Province of the
EASTERN CAPE
EDUCATION

Iphondo leMpuma Kapa: Isebe leMfundo
Provinsie van die Oos Kaap: Department van Onderwys
Provense ya Kapa Botjhabela: Lefapha la Thuto

NATIONAL SENIOR CERTIFICATE

GRADE 12

2025

REVISION MATERIAL

"It always seems impossible until it is done – N. Mandela"

MATHEMATICS

PAPER ONE & TWO

- **Practice, Practice, Practice**
  - **Attack a TOPIC at a TIME**
    - **Target LOW Hanging Fruits**
      - **Always believe in yourself**

This MANUAL consists of 132 pages, including the cover.

ALGEBRA

1. Algebraic expressions

Algebraic expressions are made up of constants, variables and number operations (add, subtract, divide and multiply).

The variables are shown with letters such as x, y, a, b, p, m, n etc.

The terms in an algebraic expression are separated by a plus sign or a minus sign.

- $2x + 3y$ is an expression with two terms which are $2x$ and $3y$.
- $2x(3y)$ is also only one term.
- $(2x + 3y)(2x - 3y)$ is also only one term because it is two expressions in brackets multiplied together. The brackets are not separated by $+$ or $-$.
- $\sqrt{2x - 3}$ is also an expression with one term because square roots can be written as exponents. $\sqrt{2x - 3} = (2x - 3)^{\frac{1}{2}}$

1.1 Addition and subtraction

- We can add or subtract like terms.
  $3x + 5x = 8x$
- If the terms are like, we add or subtract the coefficients.
  $-3a + 10a = 7a$
- Like terms have the same variables (letters) and the variables must have the same exponents.
  $6x^2y + 3x - 10x^2y = -4x^2y + 3x$
- We cannot add or subtract unlike terms.
  $\frac{a}{b} + \frac{c}{d} = \frac{ad+cb}{bd}$

1.2 Multiplication and division

- positive number × positive number = positive answer.
  $3x \times 5y^2 = 15xy^2$
- positive number × negative number = negative answer.
  $3x \times -5y^2 = -15xy^2$
- negative number × positive number = negative answer.
  $-3x \times 5y^2 = -15xy^2$
- negative number × negative number = positive answer.
  $-3x \times -5y^2 = 15xy^2$

1.3 Factorizing

What does it mean to factorize?
It means to write the expression as a product of its factors.

1. Find the **common factor**:
   $9x^2 - 6xy^2 = 3x(3x - 2y^2)$
2. Factorise by **grouping in pairs and then finding a common factor**:
   $3xy - 2x + 3y - 2$
   $= 3xy + 3y - 2x - 2$
   $= 3y(x + 1) - 2(x + 1)$
   $= (x + 1)(3y - 2)$

   When you take out a negative factor, signs in bracket change
3. Factorise the **difference of two squares**:
   $16x^2 - y^2 = (4x - y)(4x + y)$
4. Factorise the **difference of two cubes**:
   $8x^3 - y^3 = (2x - y)(4x^2 + 2xy + y^2)$
5. Factorise a **sum of two cubes**:
   $27a^3 + 64b^3 = (3a + 4b)(9a^2 - 12ab + 16b^2)$
6. Factorise a **trinomial**:
   $9x^2 + 5x - 4 = (9x - 4)(x + 1)$

HINT: **When factorizing, always take out a common factor first, if possible. Then look to factorise the difference of two squares or the sum/difference of two cubes or trinomial.**

1.4 Factorising a trinomial

Steps to factorise a trinomial;

Example 1: Factorise $3x^2 + 11x + 6$

Step 1: Multiply the coefficient of x^2 and the constant value $(+3 \times +6 = 18)$.
Step 2 :Write down all the products of 18: 18×1
9×2
6×3

Step 3: We will use 9×2, because $9 + 2 = 11$, the middle term.

Step 4: Write the middle term $11x$ as $9x + 2x$
$\therefore 3x^2 + 11x + 6$
$= 3x^2 + 11x + 6$ ……. We wrote the $9x$ first, followed by $2x$

	Step 5: We now group the four terms and factorise by taking out a common factor. $3x^2 + 9x + 2x + 6$ $= 3x(x + 3) + 2(x + 3)$ $= (x + 3)(3x + 2)$
	Example 2: Factorise $4x^2 + 9x - 13$ Step 1: Multiply the coefficient of x^2 and the constant value $(+4 \times -1 = -52)$. Step 2: Write down all the products of 52: 52×1 26×2 13×4 Step 3: We will use 13×4, because $13 - 4 = 9$, the middle term. Step 4: Write the middle term $9x$ as $-4x + 13x$ $\therefore 4x^2 + 9x - 13$ $= 4x^2 - 4x + 13x - 13$…….. we write $-4x$ first, followed by the $13x$ Step 5: We now group the four terms and factorise by taking out a common factor. $4x^2 - 4x + 13x - 13$ $= 4x(x - 1) + 13(x - 1)$ $= (x - 1)(4x + 13)$
	Example 3: Factorise $8x^2 - 18x + 9$ Step 1: Multiply the coefficient of x^2 and the constant value $(= 8 \times +9 = 72)$. Step 2: Write down all the products of 72: 72×1 36×2 24×3 18×4 12×6 9×8 Step 3: We will use 12×6, because $-12 - 6 = -18$, the middle term. Step 4: We write the middle term $(18x)$ as $-12x - 6x$ or $-6x - 12x$ $\therefore 8x^2 - 18x + 9$ $= 8x^2 - 12x - 6x + 9$……..we write the $4x$ first followed by the $13x$ Step 5: We now group the four terms and factorise by taking out a common factor. $8x^2 - 12x - 6x + 9$ $4x(2x - 3) - 3(2x - 3)$ $= (2x - 3)(4x - 3)$

2. Quadratic Equations

Here are some quadratic equations:

Example 1.

1. $x^2 + 5x + 6 = 0$
2. $3x^2 - 7x = 12$
3. $3x(x + 4) + 2 = 5x$

$3x \times x = 3x^2$

So, the equation has x^2 as its highest power of

Quadratic equations can be put in the standard form: $ax^2 + bx + c = 0$

Example 2.

1. $x^2 + 5x + 6 = 0$ $\quad a = 1,\ b = 5$ and $c = 6$
2. $3x^2 - 4x = 12$
   $3x^2 - 4x - 12 = 0$ $\quad a = 3,\ b = -4$ and $c = -12$
3. $3x(x - 9) + 2 = 5x$
   $3x^2 - 27x + 2 - 5x = 0$
   $3x^2 - 32x + 2 = 0$ $\quad a = 3,\ b = -32$ and $c = 2$

(a) Solving a quadratic equation by factorizing:

What does it mean to 'solve a quadratic equation'?

It means to find the unknown value(s) of x in a quadratic equation. The x values in a quadratic equation are also called the roots of the equation when the equation is equal to zero.

Example 3.

$x^2 - 7x = -10$
$x^2 - 7x + 10 = 0$
$x^2 - 5x - 2x + 10 = 0$
$x(x - 5) - 2(x - 5) = 0$
$(x - 5)(x - 2) = 0$
$\therefore x - 5 = 0$ or $x - 2 = 0$
$x = 5$ or $x = 2$

(b) Solving a quadratic equation by completing the square:

Example 4.

$3x^2 + 12x + 9 = 0$

$3x^2 + 12x = -9$ (1)

$\frac{3x^2}{3} + \frac{12x}{3} = -\frac{9}{3}$ (2)

Follow these steps:

1. Take the constant term to the RHS of the equation.
2. Divide each term by the coefficient of x^2
3. Take half of the coefficient of x and square the number. Add this answer both sides of the equation.
4. Now complete the square by factorizing

$x^2 + 4x = -3$

$x^2 + 4x + (+2)^2 = -3 + (+2)^2$ (3)

$x^2 + 4x + 4 = 1$

$(x + 2)(x + 2) = 1$ (4)

$(x + 2)^2 = 1$

$\sqrt{(x + 2)^2} = \pm\sqrt{1}$

$x + 2 = \pm 1$

$x = 1 - 2$ or $x = -1 - 2$
$x = -1$ or $x = -3$

(c) Solving a quadratic equation using the formula:
Some quadratic equations cannot be factorized, but there is another way to find the roots of the equation.

Example 5.
Can you find the factors for this quadratic equation: $x^2 - 5x + 3 = 0$?

There are no rational numbers that can be multiplied to get 3 and added to get 5, therefore use the quadratic formula to solve the equation.

The standard form of the quadratic equation: $ax^2 + bx + c = 0$ is used from which the formula is derived: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

For $x^2 - 5x + 3 = 0$ $a = 1, b = -5$ and $c = 3$

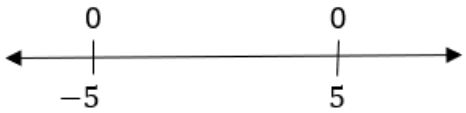
Substitute these values for a, b and c in the formula:

$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(3)}}{2(1)}$

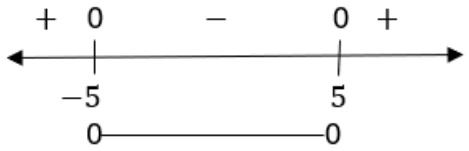
$x = \frac{5 \pm \sqrt{25 - 12}}{2}$

$x = \frac{5 \pm \sqrt{13}}{2}$

$x = \frac{5 + \sqrt{13}}{2}$ OR $x = \frac{5 - \sqrt{13}}{2}$

3.Quadratic Inequalities	Solving quadratic To solve quadratic inequalities • Get the inequality into the standard form $ax^2 + bx + c > 0$ or $ax^2 + bx + c < 0$ or $ax^2 + bx + c \leq 0$ or $ax^2 + bx + c \geq 0$ • If the value of $a < 0$, multiply the equation by -1. • Factorise inequality when this number is positive or • use the quadratic formula to obtain critical values. **Example 6.** Solve for x if $x^2 < 25$ Method 1 $x^2 < 25$ $x^2 - 25 < 0$ $(x + 5)(x - 5) < 0$ The critical values are where the expression $x^2 - 25$ is equal to zero. Therefore, the critical values are -5 and 5. Now indicate -5 and 5 on the number line.  We know that the expression $x^2 - 25 = 0$ at -5 and 5. We can indicate this on the number line.  Our next step is to choose values less than -5, values between -5 and 5 and value greater than 5 and substitute it into the expression $x^2 - 25$. If the answer is positive, then we indicate $+$ on the number line. If the answer is negative, we indicate $-$ on the number line. 

<table>
<tr><td></td><td colspan="2">We must solve for x where $x^2 - 25 < 0$. The solution on the number line is the interval where we see a negative. This happens between -5 and 5.

Therefore, the solution is : $-5 < x < 5$</td></tr>
<tr><td>4. Simultaneous equations</td><td>Example 7

Solve for x and y simultaneously:
$y + 2x - 2 = 0$ and $2x^2 + y^2 = 3xy$

In this example, a quadratic equation and a linear equation must be solved simultaneously. Use the following steps:

Step 1: Use the linear equation to make one of unknowns the subject of the equation.

Step 2: Substitute x or y into the quadratic equation. The equation will contain only one unknown.

Step 3: Solve the one unknown.

Step 4: Substitute the solved unknown into the linear equation to solve the other unknown.</td><td>Solution

$y + 2x - 2 = 0$ ……..eqn (1)
$2x^2 + y^2 = 3xy$ …….eqn (2)

Step 1: $y + 2x - 2 = 0$ from eqn (1)
$\therefore y = 2 - 2x$ …… eqn (3)

Step 2: Substitute eqn (3) in eqn (2)
$2x^2 + y^2 = 3xy$
$\therefore 2x^2 + (2 - 2x)^2 = 3x(2 - 2x)$

Step 3: $2x^2 + (2 - 2x)(2 - 2x) = 3x(2 - x)$
$2x^2 + 4 - 8x + 4x^2 = 6x - 6x^2$
$12x^2 - 14x + 4 = 0$
$\div 2 \ \therefore 6x^2 - 7x + 2 = 0$
$\therefore (3x - 2)(2x - 1) = 0$
$\therefore x = \frac{2}{3}$ or $x = \frac{1}{2}$

Step 4: Substitute $x = \frac{2}{3}$ in eqn (3)
$\therefore y = 2 - 2(\frac{2}{3})$
$\therefore y = \frac{2}{3}$
Substitute $x = \frac{1}{2}$ in eqn (3)
$\therefore y = 2 - 2(\frac{1}{2})$
$\therefore y = 1$</td></tr>
<tr><td>5. The Nature of the roots</td><td colspan="2">The roots of any quadratic equation $ax^2 + bx + c = 0$ can be found by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

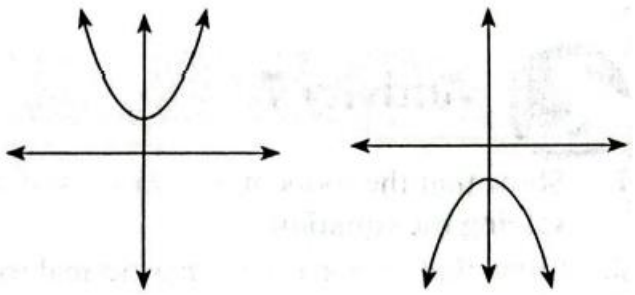
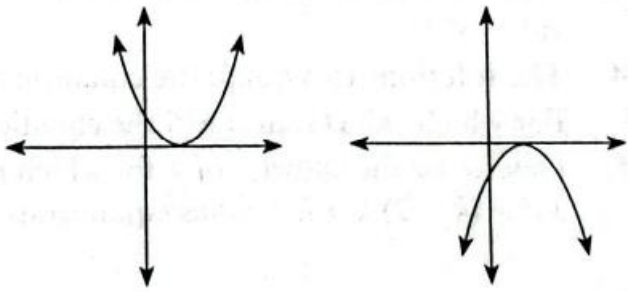
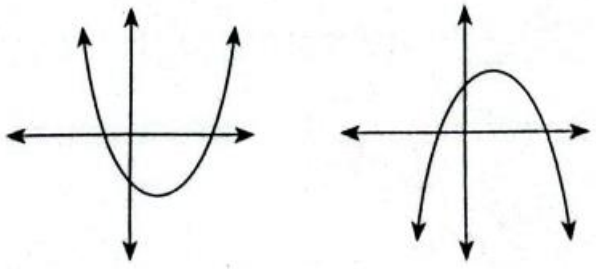
• The roots of a quadratic equation are the x-values when the equation is zero.
• The roots are the x-intercept of the graph.</td></tr>
</table>

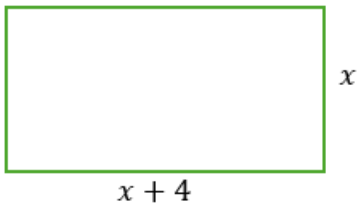
- When you are asked to 'determine the nature of the roots of an equation', you are NOT asked to solve the equation.

Summary:

To find the nature of the roots of a quadratic equation $ax^2 + bx + c = 0$, look at the value of D, the discriminant. ($\Delta = b^2 - 4ac$)

- If $\Delta < 0$: The roots are non-real/ no real roots.
- If $\Delta = 0$: There are two equal, real and rational roots.
- If $\Delta > 0$: There are two real roots which may be rational or irrational.
  If D is a perfect square, the roots are rational.
  If D is not a perfect square, then the roots are irrational.

Nature of the roots	**Graphs**
$\Delta < 0$ Roots are non-real. There are no x-intercepts.	
$\Delta = 0$ Roots are real and equal There is only one x-intercept and it is the turning point of the graph.	
$\Delta > 0$ Roots are real and unequal (two roots): If Δ is a squared rational number, roots are rational. If Δ is not a squared number, the roots are irrational.	

6. Problem solving using quadratic equations	You can use an equation to represent a problem. Find what part of the problem is unknown and needs to be represented by a variable. **Example 8.** The are of a rectangle is $12\ m^2$. The length is 4 metres longer than the breadth. Find the dimensions of the rectangle. We don't know the length of the breadth of the rectangle. We do know that the length is 4 metres longer than the breadth. It makes sense to let the breadth be x metres. Then the length is $x + 4$ metres.
	Draw a picture to help you. Let breadth be x metres.  Area of rectangle = length × breadth $12 = (x + 4)x$ $12 = x^2 + 4x$ $0 = x^2 + 4x - 12$ $0 = (x + 6)(x - 2)$ $\therefore x + 6 = 0$ or $x - 2 = 0$ $x = -6$ or $x = 2$ Length and breadth must be positive lengths. You can't have a negative length! So $x \neq -6$ $\therefore x = 2$ and so, the breadth is 2 metres. The length is $x + 4$ and so the length is 6 metres.

SOURCE

Mind the Gap Mathematics Study Guide Grade 12

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COMPILATION

OF

QUESTION BY QUESTION

(NSC & SC 2022 – 2025)

MATHEMATICS P1

GRADE 12

ALGEBRA, EQUATIONS AND INEQUALITIES

QUESTION 1

1.1 Solve for x: ***SC/May/June 2025***

1.1.1 $x^2 - 3x - 10 = 0$ (3)

1.1.2 $3x^2 + 6x + 1 = 0$ (correct to TWO decimal places) (3)

1.1.3 $2^{x+4} + 2^x = 8\,704$ (3)

1.1.4 $(x - 8)(x + 2) \leq 0$ (3)

1.1.5 $x + 3\sqrt{x + 2} = 2$ (4)

1.2 A rectangle having sides of $(y - 3)$ metres and $(x + 2)$ metres has perimeter of 24 metres and an area of 32 square metres.
Calculate the value of x and y. (6)

1.3 Solve for x : ***NSC/November 2022***

1.3.1 $(3x - 6)(x + 2) = 0$ (2)

1.3.2 $2x^2 - 6x + 1 = 0$ (correct to TWO decimal places) (3)

1.3.3 $x^2 - 90 > x$ (4)

1.3.4 $x - 7\sqrt{x} = -12$ (4)

1.4 Solve for x and y simultaneously:

$2x - y = 2$
$xy = 4$ (5)

1.5 Solve for x, in each of the following ***EC/June 2022***

1.5.1 $x^2 = -4x$ (3)

1.5.2 $x^2 + x - 1 = 0$ (correct to TWO decimal places) (3)

1.5.3 $\sqrt{x + 4} - \dfrac{4}{\sqrt{x - 2}} = 0$ (5)

1.5.4 $(x + 2)(-3x + 1) > 0$ (3)

1.6 Solve for x and y simultaneously:

$3 - y + 2x = 0$
$6x + 4y^2 = 3 + 5xy$ (6)

1.7 Given that $9x^2 - 12px = -4p^2$. For which values of p will the equation have equal roots? (4)

1.8 Solve for x: ***EC/June 2024***

1.8.1 $x^2 - 8(x - 2) = 25$ (3)

1.8.2 $-3x^2 + 2x + 2 = 0$ (correct to TWO decimal places) (3)

1.8.3 $(x + 3)(5 - x) \leq 0$ (3)

1.8.4 Given: $\frac{x+3}{\sqrt{x+5}} = 1$; $x \in \mathbb{R}$

(a) For which value(s) of x will $\frac{x+3}{\sqrt{x+5}}$ be undefined? (2)

(b) Solve for x. (4)

1.9 Solve simultaneously for x and y:

$y + 2x = 5$
$2x^2 - xy - 4y^2 = 8$ (6)

1.10 Solve for x: ***SC/May/June 2023***

1.10.1 $x^2 - 7x + 12 = 0$ (3)

1.10.2 $x(3x + 5) = 1$ (correct to TWO decimal places) (4)

1.10.3 $x^2 < -2x + 15$ (4)

1.10.4 $\sqrt{2(1 - x)} = x - 1$ (4)

1.11 Solve for x and y simultaneously:

$3^{x+y} = 27$ and $x^2 + y^2 = 17$ (6)

1.12 Solve for x: ***NSC/November 2024***

1.12.1 $x(x - 3) = 0$ (2)

1.12.2 $2x^2 + 1 = 4x$ (correct to TWO decimal places) (4)

1.12.3 $x^2 - 2x - 3 = 0$ (4)

1.12.4 $2^{2x} - 2^{x+2} - 32 = 0$ (5)

1.12.5 $\sqrt{-2x+4} - x = 2$ (4)

1.13 Solve for x and y simultaneously:

$2x + y = 3$
$y^2 + xy = 2$ (5)

1.14 Solve for x: ***NSC/November 2023***

1.14.1 $x^2 + x - 12 = 0$ (3)

1.14.2 $3x^2 - 2x = 6$ (4)

1.14.3 $\sqrt{2x+1} = x - 1$ (4)

1.14.4 $x^2 - 3 > 2x$ (4)

1.15 Solve for x and y simultaneously:

$x + 2 = 2y$ and $\frac{1}{x} + \frac{1}{y} = 1$ (5)

1.16 Solve for x : ***SC/May/June 2024***

1.16.1 $3x^2 + 5x = 0$ (2)

1.16.2 $4x^2 + 3x - 5 = 0$ (correct to TWO decimal places) (3)

1.16.3 $(x-1)^2 - 9 \geq 0$ (4)

1.16.4 $5^{2x} - 5^x = 0$ (4)

1.16.5 $\frac{x}{\sqrt{20-x}} = 1$ (5)

1.17 Solve for x and y simultaneously:

$x + y = 9$ and $2x^2 - y^2 = 7$ (5)

1.18 Solve for x: ***SC/May/June 2022***

1.18.1 $x^2 + 2x - 15 = 0$ (3)

1.18.2 $5x^2 - x - 9 = 0$ (correct to TWO decimal places) (3)

1.18.3 $x^2 \leq 3x$ (4)

1.19 Given: $a + \frac{64}{a} = 16$

1.19.1 Solve for a (3)

1.19.2 Hence, solve for x if: $2^x + 2^{6-x} = 16$ (3)

1.20 Without using a calculator, calculate the value of: $\sqrt{\frac{2^{1002} + 2^{1006}}{17(2)^{998}}}$ (4)

1.21 Solve for x and y simultaneously:

$2x - y = 2$ and $\frac{1}{x} - 3y = 1$ (6)

NUMBER PATTERNS AND SEQUENCES

QUESTION 2

NSC/November 2023

2.1 Given the arithmetic series: $7 + 12 + 17 + \ldots$

2.1.1 Determine the value of T_{91} (3)

2.1.2 Calculate S_{91} (2)

2.1.3 Calculate the value of n for which $T_n = 517$ (3)

2.2 The following information is given about a quadratic number pattern:

$T_1 = 3$, $T_2 - T_1 = 9$, and $T_3 - T_2 = 21$

2.2.1 Show that $T_5 = 111$ (2)

2.2.2 Show that the general term of the quadratic pattern is $T_n = 6n^2 - 9n + 6$ (3)

2.2.3 Show that the pattern is increasing for all $n \in N$. (3)

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2.3 Given the following arithmetic sequence: $2;\ -3;\ -8;\ \ldots..$

2.3.1 Determine the value of T_{43} (3)

2.3.2 Calculate the sum of the first 43 terms in the row, i.e. S_{43} (2)

2.3.3 Calculate the value of n for which $T_n = -2023$. (3)

2.4 Given: $2(3x - 1) + 2(3x - 1)^2 + \ldots.$

2.4.1 For which values of x is the series above a convergent geometric series? (3)

2.4.2 Calculate $\sum_{k=1}^{\infty} 2(3x - 1)^k$; if $x = \frac{1}{2}$ (3)

2.5 The first three terms of a geometric sequence has a sum of 21 and their product is 64. Determine the value of the first term, if the common ration is an integer, i.e. $r \in Z$. (4)

2.6 Consider the following quadratic number pattern: 3 ; 12 ; 33 ; . . .

2.6.1 Write down the next term in the quadratic number pattern. (1)

2.6.2 Determine the general term of the quadratic number pattern in the form $T_n = an^2 + bn + c$. (3)

2.6.3 Which TWO terms in the quadratic number pattern will have a difference of 345? (3)

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2.7 Given the geometric sequence: $\frac{9}{2}$; 9 ; 18 ; . . . ; 2304.

2.7.1 Determine the value of r, the common ratio. (1)

2.7.2 How many terms are there in the sequence? (2)

2.8 Given: $\sum_{k=1}^{\infty} 6(m)^{k-1} = 12$. Determine the value of m. (2)

2.9 The 3rd term of a geometric series is 18 and the 5th term is 162. Determine the sum of the first 7 terms, where $r < 0$. (5)

2.10 The general term of a quadratic number pattern is $T_n = an^2 + bn + c$ and its first term is 8.
The general term of the first differences of the pattern is $t_k = 4k - 2$.

2.10.1 Determine the next two terms of the quadratic number pattern, T_n . (3)

2.10.2 Hence, or otherwise, show that the general term of the quadratic number pattern is given by $T_n\ 2n^2 - 4n + 10$. (3)

2.10.3 Which term of the quadratic number pattern will be equal to 3 050 ? (3)

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2.11 The first term of a geometric series is 14 and the 6th term is 448.

2.11.1 Calculate the value of the constant ratio, r . (2)

2.11.2 Determine the number of consecutive terms that that must be added to the first 6 terms of the series in order to obtain a sum of 114 674. (4)

2.11.3 If the first term of another series is 448 and the 6th term is 14, calculate the sum to infinity of the new series. (3)

2.12 If $\sum_{p=0}^{k} \left(\frac{1}{3}p + \frac{1}{6}\right) = 20\frac{1}{6}$, determine the value of k. (5)

2.13 It is given that the general term of a quadratic number pattern is $T_n = n^2 + bn + 9$, And the first term of the first differences is 7.

2.13.1 Show that $b = 4$. (2)

2.13.2 Determine the value of the 60th term of this number pattern. (2)

2.13.3 Determine the general term for the sequence of first differences of the quadratic number pattern. Write your answer in the form $T_p = mp + q$. (3)

2.13.4 Which TWO consecutive terms in the quadratic number pattern have a first difference of 157? (3)

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2.14 The first term of an arithmetic sequence is -1 and the 7th term is 35.

Determine:

2.14.1 The common difference of the sequence (2)

2.14.2 The number of terms in the sequence if the last term of the sequence is 473 (3)

2.14.3 The sum of the first 40 terms in this sequence (2)

2.15 75 ; 53 ; 35 ; 21; . . . is a quadratic number pattern.

2.15.1 Write down the FIFTH term of the number pattern. (1)

2.15.2 Determine the n^{th} term of the number pattern. (4)

2.16 Consider the following geometric sequence: 1 024 ; 256 ; 64 ; . . .

Calculate:

2.16.1 The 10th term of the sequence (2)

2.16.2 $\sum_{p=0}^{8} 256(4^{\,1-p})$ (4)

2.17 The first two terms of a geometric sequence are:

$-t^2 - 6t - 9$ and $\dfrac{t^3 + 9t^2 + 27t + 27}{2}$

Determine the values of t for which the sequence will converge. (5)

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2.18 The general term of a geometric sequence is $T_n = 2^{n+2}$

2.18.1 Write down:

(a) The first term (1)

(b) The common ratio (1)

2.18.2 Calculate T_{20} (Write your answer as a power of 4.) (2)

2.18.3 Calculate $\sum_{n=1}^{\infty} \dfrac{1}{T_n}$ (3)

2.18.4 Consider the first 21 terms of the sequence $T_n = 2^{n+2}$.
Calculate the sum of the terms in this sequence that are not powers of 4. (4)

2.19 Given the quadratic sequence: 14 ; 9 ; 6 ; 5 ; ...

2.19.1 Show that the general term of this sequence is $T_n = n^2 - 8n + 21$. (3)

2.19.2 Two consecutive terms of the quadratic sequence have a difference of 33.
Calculate the value of the larger term. (3)

2.19.3 The value of m is added to each term in the quadratic sequence. Determine the values of m for which only the terms between T_1 and T_7 of the quadratic sequence will have negative values. (3)

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2.20 Consider the geometric series: $4 + 2 + 1 + \frac{1}{2} + \ldots$.

2.20.1 Does this series converge? Justify your answer. (2)

2.20.2 Calculate S_∞ (2)

2.21 Given: $\sum_{p=k}^{10} 3^{p-1} = 29\,520$. Calculate the value of k. (5)

2.22 Consider the quadratic number pattern: 3 ; 7 ; 12 ; ...

2.22.1 Show that the general term of this number pattern is given by
$T_n = \frac{1}{2}n^2 + \frac{5}{2}n$. (3)

2.22.2 What number must be added to T_{n-1} so that $T_n = 13\,527$? (4)

2.23 Given an arithmetic sequence with $T_1 = 8$ and $T_2 = 11$.

2.23.1 Calculate the value on n if $T_n = 41$. (3)

2.23.2 A new arithmetic sequence P is formed using the term position and the term value of the given arithmetic sequence.
For the new sequence, $P_8 = 1$, $P_{11} = 2$ and so forth.

(a) Write down the value of P_{41} (1)

(b) Calculate the value of the first term of term of the new arithmetic (4)

2.24 Given the geometric series: $\frac{1}{5} + \frac{1}{15} + \frac{1}{45} + \ldots$

2.24.1 Is this a convergent geometric series? Justify your answer with the necessary calculations. (2)

2.24.2 Calculate the sum to infinity of this series. (2)

2.25 An arithmetic and geometric sequence are combined to form the pattern which is given by : $P_n = x \ ; \ \frac{1}{3} \ ; \ 2x \ ; \ \frac{1}{9} \ ; \ 3x \ ; \ \frac{1}{27} \ ; \ \ldots$

2.25.1 Write down the next TWO terms of the pattern. (2)

2.25.2 Determine the general term (T_n) for the odd terms of this pattern. Write down your answer in terms of x. (2)

2.25.3 Calculate the value of P_{26}. (3)

2..25.4 If $\sum_{n=1}^{21} P_n = 33{,}5$. Determine the value of x. (6)

2.26 A quadratic sequence has the following properties:

- The second difference is 10.
- The first two terms are equal, i.e. $T_1 = T_2$.
- $T_1 + T_2 + T_3 = 28$

2.26.1 Show that the general term of the sequence is $T_n \ 5n^2 - 15n + 16$. (6)

2.26.2 Is 216 a term in this sequence? Justify your answer with the necessary calculations. (3)

GRAPHS, FUNCTIONS and INVERSES

QUESTION 3

NS/May/June 2024

3.1 Given: $g(x) = \frac{1}{x-1} + 2$

3.1.1 Write down the equations of the asymptotes of g. (2)

3.1.2 Draw the graph of g, indicating any intercepts with the axes and asymptotes. (4)

3.1.3 Determine the values of x where $g(x) > 0$. (2)

3.1.4 Determine the equation of the axis of symmetry of g which has negative gradient. (2)

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3.2 Given: $f(x) = \frac{8}{x-2} + 2$

3.2.1 Write down the domain of f. (2)

3.2.2 Calculate the $y-$ intercept of f. (1)

3.2.3 Calculate the $x-$ intercept of f. (2)

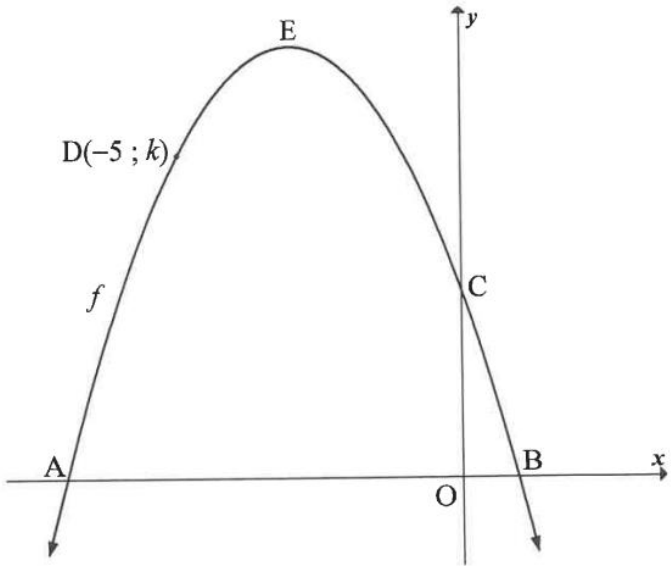
3.2.4 Sketch the graph of f, clearly indicating the coordinates of the x and $y-$ intercept of f. as well as the asymptotes. (3)

3.2.5 If $y = -x + k$ is an equation of the line of symmetry of f, determine the value of k. (2)

3.2.6 Determine the equation of the graph formed if f is shifted 3 units to the right and then reflected across the $x-$axis. (3)

3.3 The sketch below shows the graph of $f(x) = -x^2 - 6x + 7$.

- C is the y −iintercept of f.
- A and B are the x −intercepts of f.
- D$(-5\ ;k)$ is a point on f.



3.3.1 Calculate the coordinate of E, the turning point of f. (3)

3.3.2 Write down the value of k. (1)

3.3.3 Determine the equation of the straight line passing through C and D. (4)

3.3.4 A tangent parallel to CD, touches f at P. Determine the coordinates of P. (4)

3.3.5 For which values of x will $f(x) - 12 > 0$? (2)

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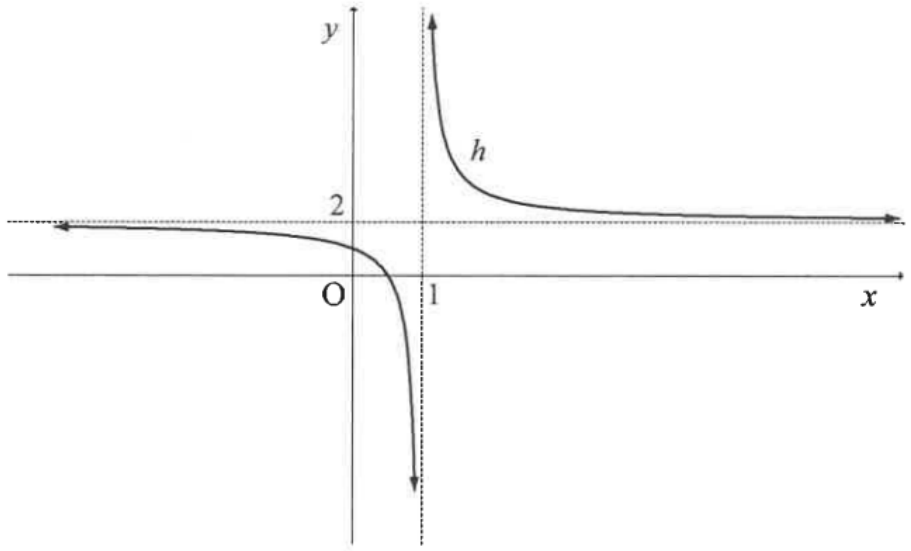
3.4 Sketched below is the graph of $h\ (x) = \frac{1}{x-p} + q$. The asymptotes of h intersect at $(1\ ;\ 2)$.



3.4.1 Write down the values of p and q. (2)

3.4.2 Calculate the coordinates of the x −intercept of h. (2)

3.4.3 Write down the x −coordinates of the x −intercept of g if $g\ (x) = h\ (x + 3)$. (2)

3.4.4 The equation of an axis of symmetry of h is $y = x + t$. Determine the value of t. (2)

3.4.5 Determine the values of x for which $-2 \leq \frac{1}{x-1}$. (3)

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3.5 Given: $f(x) = a^x - 1$ for $a > 0$. B$\left(2\ ;\ -\frac{5}{9}\right)$ is a point on f.

3.5.1 Calculate the value of a. (2)

3.5.2 Write down the range of f. (1)

3.5.3 Sketch the graph of f. Clearly show the intercepts with the axes and asymptotes, if any. (3)

3.5.4 It is further that C is a point on f at $y = \frac{19}{8}$.

Determine the coordinates of C' , the image of C, when C is reflected about the line $y = x$. (3)

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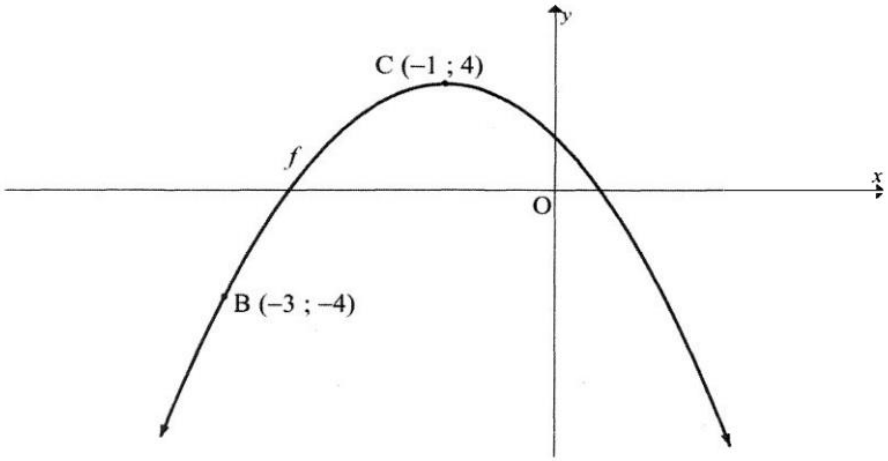
3.6 The graph of $f(x) = a(x+p)^2 + q$ is drawn below. C $(-1\,;4)$ is the turning point of f. B$(-3\,;\,-4)$ is a point on f.



3.6.1 Show that $f(x) = -2x^2 - 4x + 2$. (3)

3.6.2 Determine the values of k for which $h(x) = f(x) + k$ will have no real roots. (2)

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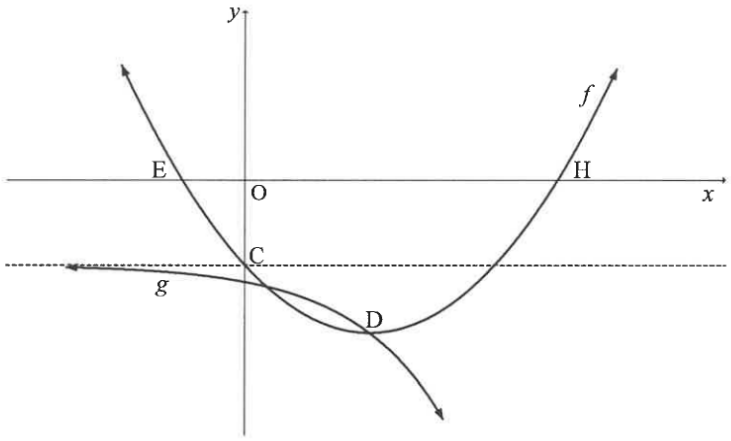
3.7 The graphs of $f(x) = x^2 - 4x - 5$ and $g(x) = a.2^x + q$ are sketched below.

- E and H are the x −intercepts of f.
- C is the y −intercept of f and lies on the asymptotes of g.
- The two graphs intersect a D, the turning point of f.



3.7.1 Write down the y −intercept of C. (1)

3.7.2 Determine the coordinates of D. (2)

3.7.3 Determine the values of a and q. (3)

3.7.4 Write down the range of g. (1)

3.7.5 Determine the values of k for which the value of $f(x) - k$ which will Always be positive. (2)

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3.8 The graph of $g(x) = a\left(\frac{1}{3}\right)^x + 7$ passes through point E $(-2\ ;\ 10)$.

3.8.1 Calculate the value of a. (3)

3.8.2 Calculate the coordinates of the y −intercept of g. (2)

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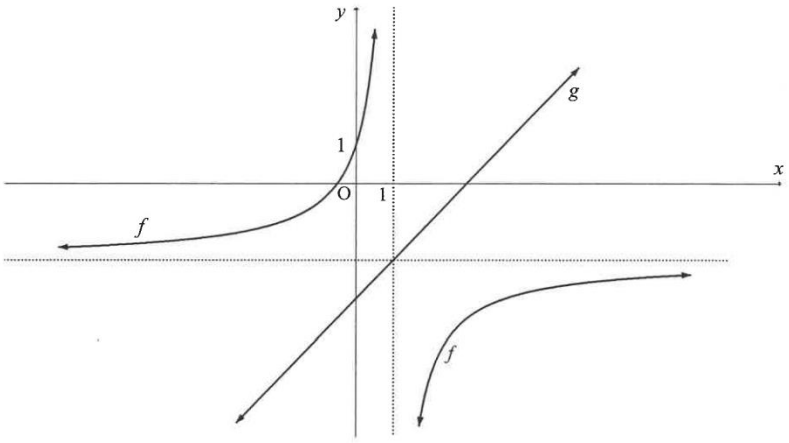
3.9 Sketched below is the graph of $f(x) = \frac{a}{x+p} + q$ having the domain $(-\infty\ ;1) \cup (1\ ;\ \infty)$. The graph of f cuts the y −axis at $(0\ ;\ 1)$. A line of symmetry of f is given by $g(x) = x - 3$.



3.9.1 Write down the value of p. (1)

3.9.2 Determine the equation of the horizontal asymptote of f. (2)

3.9.3 Calculate the value of a. (2)

3.9.4 For which values of x is $f(x) = \geq 0$? (3)

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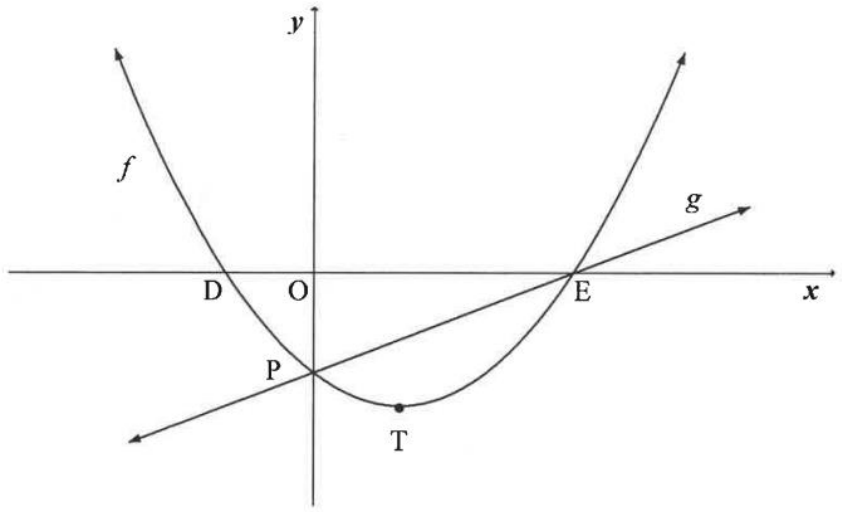
3.10 The graphs of $f(x) = x^2 - 2x - 3$ and $g(x) = mx + c$ are drawn below. D and E are the $x-$ intercepts and P is the $y-$ intercept of f. The turning of f is T $1 ; -4$).
The graphs of f and g intersect at P and E.



3.10.1 Write the range of f. (1)

3.10.2 Calculate the coordinates of D and E. (3)

3.10.3 Determine the equation of g. (2)

3.10.4 Write down the values of x for which $f(x) - g(x) > 0$. (2)

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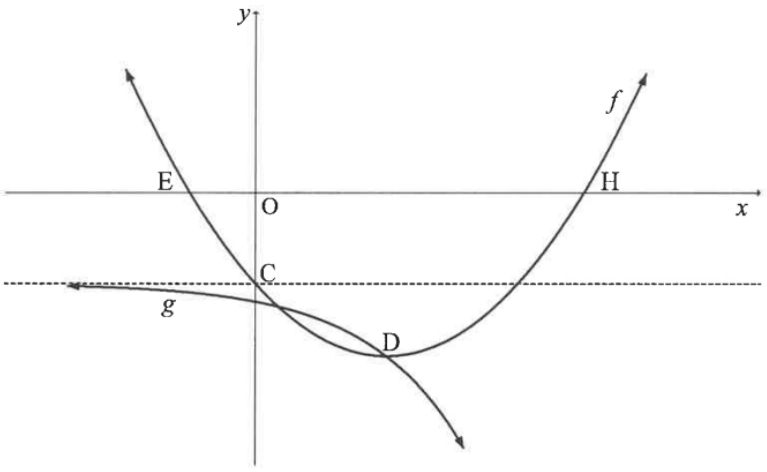
3.11 The graphs of $f(x) = x^2 - 4x - 5$ and $g(x) = a.2^x + q$ are sketched below.

- E and H are the $x-$ intercepts of f.
- C is the $y-$ intercept of f and lies on the same asymptote of g.
- The two graphs intersect at D, the turning point of f.



3.11.1 Write down the $y-$coordinate of C. (1)

3.11.2 Determine the coordinates of D. (2)

3.11.3 Determine the values of a and q. (3)

3.11.4 Write down the range of g. (1)

3.11.5 Determine the values of k for which the value of $f(x) - k$ will always be positive. (2)

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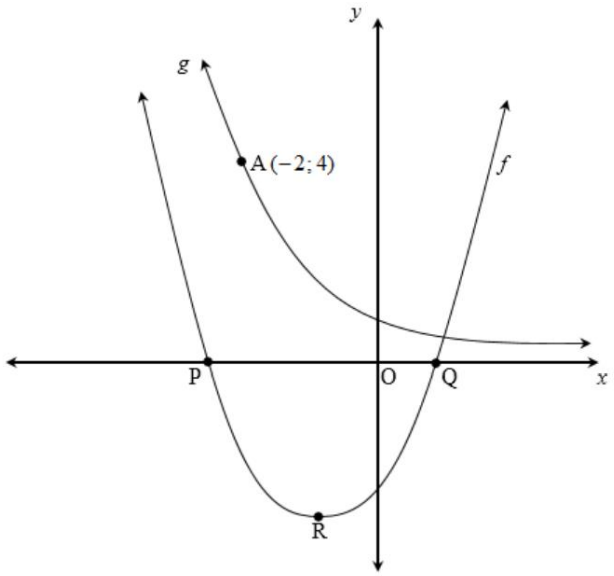
3.12 The graphs of $f(x) = 2(x+1)^2 - 8$ and $g(x) = \left(\frac{1}{2}\right)^x$ are represented in the sketch below. P and Q are the $x-$ intercepts of f and R is the turning point f.
A$(-2\,;4)$ is a point on the graph of g.



3.12.1 Write down the equation of the axis of symmetry of f. (1)

3.12.2 Write down the coordinates of R, the turning point of f. (1)

3.12.3 Determine the coordinates of P and Q. (4)

3.12.4 Determine the equation of g^{-1}, the inverse of g, in the form $y = \ldots$. (2)

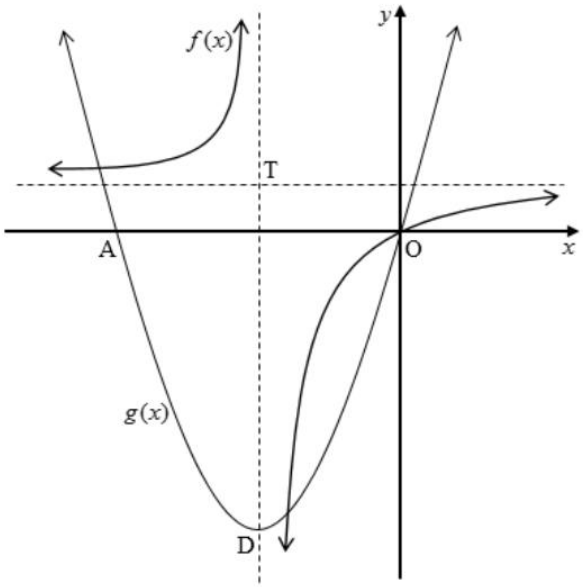
3.12.5 Sketch the of g^{-1} in your ANSWER BOOK. Clearly indicate the intercept with the axis and at least ONE other point on g^{-1}. (3)

3.12.6 (a) $g^{-1} \geq -2$? (2)

(b) $x \,.\, f(x) < 0$? (3)

3.13 The diagram below shows the graphs of $f(x) \frac{2}{x+2} + 1$ and $g(x) = a(x+2)^2 - 8$.

Both graphs pass through the origin, O. The vertical asymptote of *f* passes through D, the turning point of g. The asymptotes of *f* intersect at T.
A is the other *x*-intercept of *g*.



3.13.1 Write down the coordinates of D, the turning point of *g*. (1)

3.13.2 Write down the equations of the asymptotes of *f*. (2)

3.13.3 Determine:

3.13.1 The value of *a* (2)

3.13.2 The length OA (3)

3.13.3 The range of f (1)

3.13.4 The equation of the axis of symmetry of *f* with a negative gradient ((2)

3.13.4 For which values of *x* will:

(a) $g(x) < 0$? (2)

(b) $g(x) \cdot f(x) \ 0 \geq$? (2)

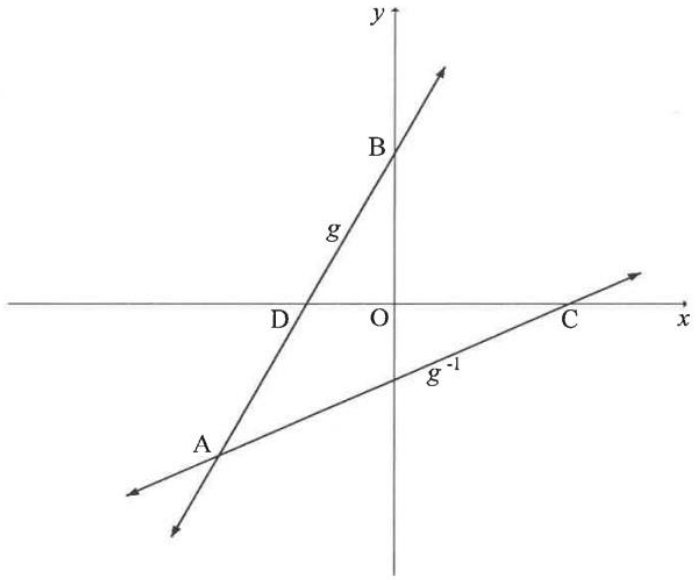
3.13.5 Determine the value(s) of *k*, for which $h(x) = -g(x) + k$ will have two distinct roots with the same sign. (3)

QUESTION 4

NSC/November 2022

4.1 The graphs of $g(x) = 2x + 6$ and g^{-1}, the inverse of g, are shown in the diagram below.

- D and B are the $x-$ and $y-$ intercepts respectively of g.
- C is the $x-$ intercept of g^{-1}.
- The graphs of g and g^{-1} intersect at A.



4.1.1 Write down the $y-$coordiate of B. (1)

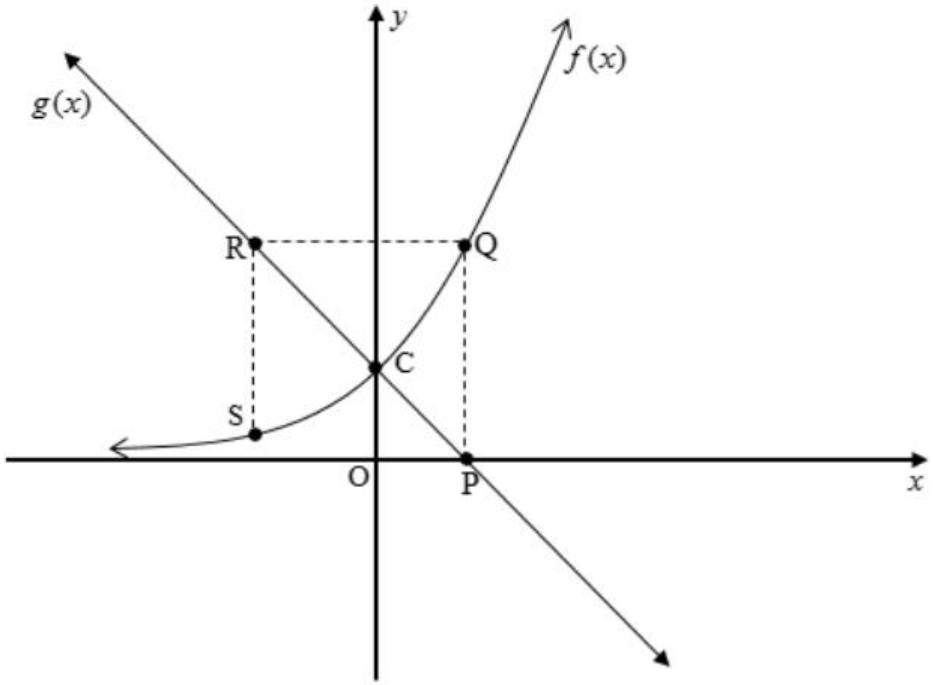
4.1.2 Determine the equation of g^{-1} in the form $g^{-1}(x) = mx + n$. (2)

4.1.3 Determine the coordinates of A. (3)

4.1.4 Calculate the length of AB. (2)

4.1.5 Calculate the area of Δ ABC. (5)

4.2 In the diagram below, the graphs of $f(x) = 3^x$ and $g(x) = -x + 1$ are given.



4.2.1 Write down the coordinates of C. (1)

4.2.2 Write down the range of $f(x)$. (1)

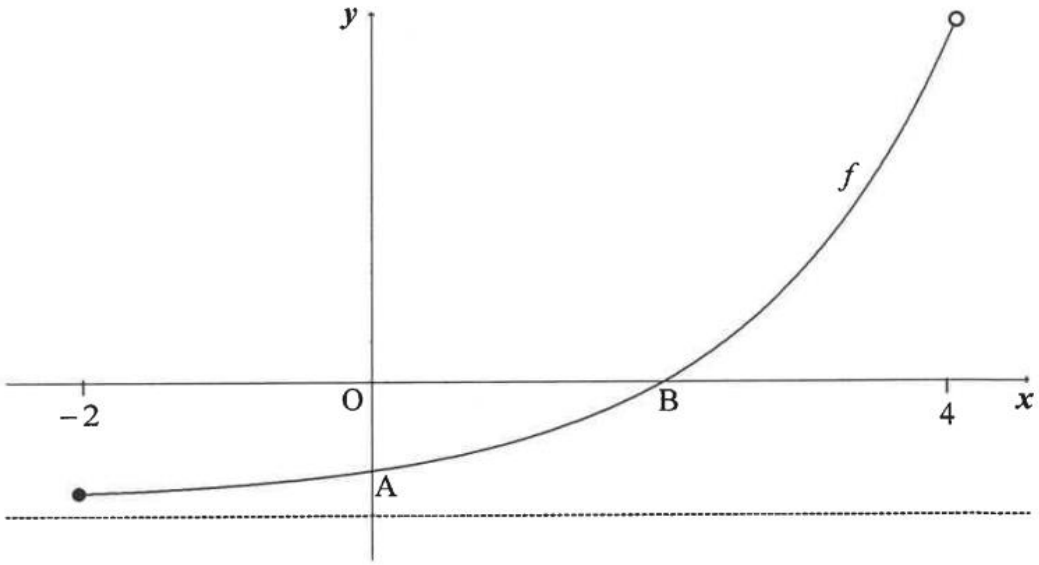
4.2.3 Determine the equation of f^{-1} (x), in the form $y = ...$ (2)

4.2.4 For which values of x is $f^{-1}(x) < -1$ (2)

4.2.5 If PQ || SR || y-axis and QR ∥ x-*axis*, determine the coordinates of S. (4)

4.2.6 Describe the translation(s) of $f(x)$ to $p(x) = 3(3^x) - 2$ (2)

4.3 Sketched below is the graph of $f(x) = 2^x - 4$ for $x \in [-2 ; 4)$.
A and B are the $y-$ and $x-$ intercepts of f respectively.



4.3.1 Write down the equation of the asymptote of f. (1)

4.3.2 Determine the coordinates of B. (2)

4.3.3 Determine the equation of k, a straight line passing through A and B in the form of $k(x) = \ldots$ (3)

4.3.4 Calculate the vertical distance between k and f at $x = 1$ (3)

4.3.5 Write down the equation of g if it is given that $g(x) = f(x) + 4$ (1)

4.3.6 Write down the domain of g^{-1}. (2)

4.3.7 Write down the equation of g^{-1} in the form $y = \ldots$ (2)

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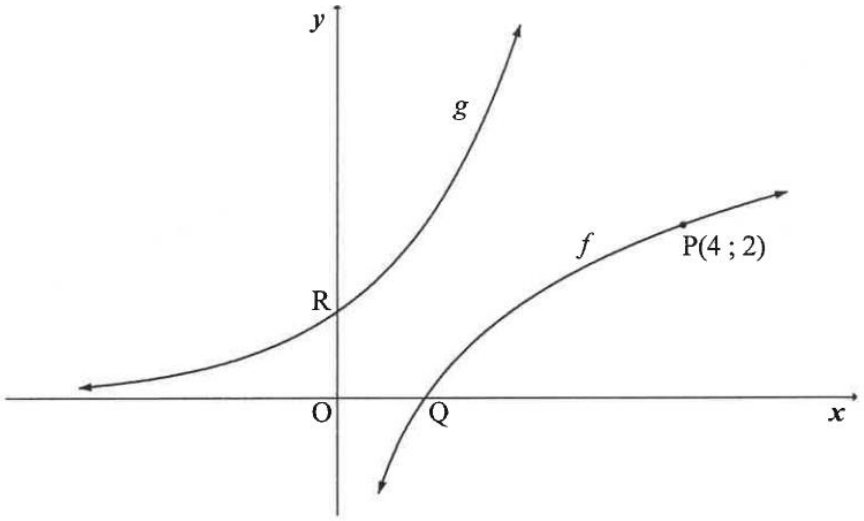
4.4 In the diagram below, the graphs of $f(x) = log_a\, x$ and g are drawn. Graph of g is the reflection of f in the line $y = x$. Graph of f passes through the point P(4 ; 2). Q is the $x-$ intercepts of f and R is the $y-$ intercept of g.



4.4.1 Write down the coordinates of $P^{/}$, the image of P on g. (2)

4.4.2 Show that $a = 2$ (2)

4.4.3 Write down the equation of g in the form of $y = \ldots$ (1)

4.4.4 T is a point on f in the first quadrant where TR is parallel to the x-axis . Calculate the area of $\Delta RTP^{/}$. (4)

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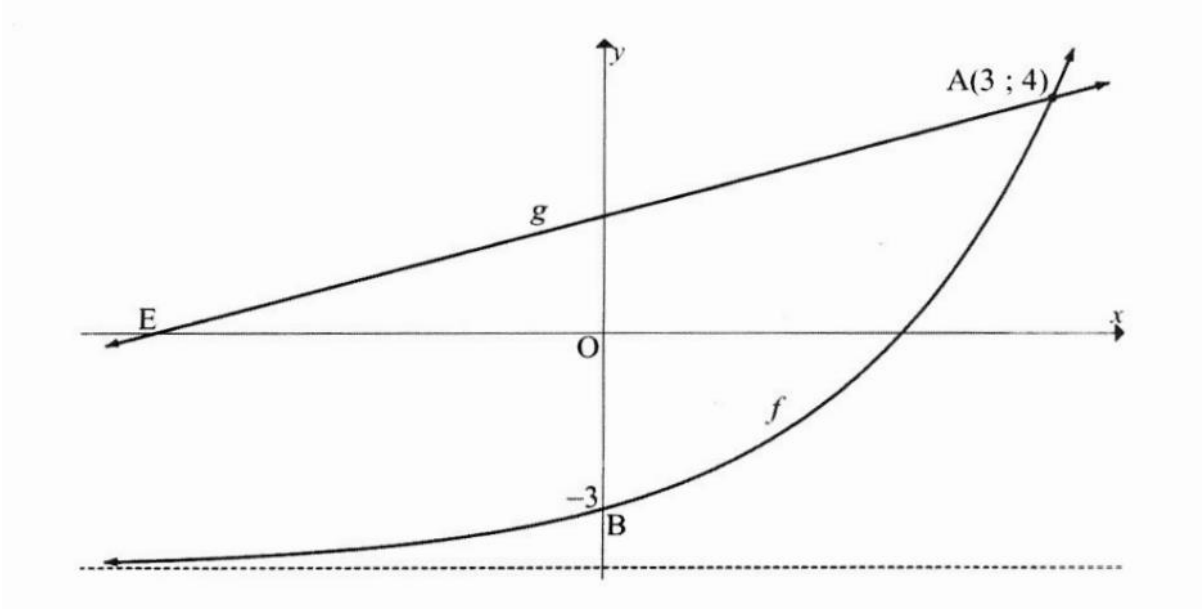
4.5 The graphs of $f(x) = p^x + q$ and $g(x) = mx + c$ are drawn below.
A (3 ; 4) is the point of intersection of f and g . B(0 ; -3) is the $y-$ intercept of f.
E is the x-intercept of g.



4.5.1 Calculate the values of p and q. (4)

4.5.2 Write down the range of f. (1)

4.5.3 The graph of g^{-1}, the inverse of g, also passes through B. Determine the equation of g in the form $y = \ldots$ (4)

4.5.4 Write down the equation of g^{-1} in the form $y = \ldots$ (2)

FINANCE, GROWTH AND DECAY

QUESTION 5

NSC/November 2022

5.1 R12 000 was invested in a fund that paid interest at m % p.a., compounded quarterly. After 24 months, the value of the investment was R13 459.

Determine the value of m. (4)

5.2 On 31 January 2022, Tino deposited R1 000 in an account that paid interest at 7,5 % p.a., compounded monthly. He continued depositing R1 000 on the last day of every month. He will make the last deposit on 31 December 2022.

Will Tino have sufficient funds in the account on 1 January 2023 to buy a computer that costs R13 000? Justify your answer by means of an appropriate calculation. (4)

5.3 Thabo plans to buy a car that costs R250 000. He will pay a deposit of 15% and take out a loan for the balance. The interest on the loan is 13% p.a., compounded monthly.

5.3.1 Calculate the value of the loan. (1)

5.3.2 The first repayment will be made 6 months after the loan has been granted. The loan will be repaid over a period of 6 months after it has been granted. Calculate the MONTHLY instalment. (5)

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5.4 How long must R50 000 be invested, in order for it to double at an interest rate of 8,5% p.a. on the straight-line method? (Give your answer in years and months.) (4)

5.5 A cellphone valued at R24 000 depreciates at 18% p.a. on the reducing balance method. Determine the value of the cellphone after 3 years. (3)

5.6 R x (x Rand) is invested into an account at an interest rate of 12% p.a. compounded monthly. Three years later R2x (2x Rand) is deposited into the same account. After 7 years there is R276 558,75 in the account. Determine how much money was invested at the beginning. (that is, the value of x) (6)

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5.7 The purchase price of machinery bought by a company 5 years ago was R80 000. Using the reducing-balance method, calculate the annual rate of depreciation if the current book value of the machinery is R20 000. (3)

5.8 Calculate the effective interest rate per annum of an investment earning interest at 8,5% p.a. compounded quarterly. (3)

5.9 A parent made an initial deposit of R x into a study investment account. Three years later a further amount of R15 000 is deposited into the account. Five years after the initial deposit was made, R7 000 was withdrawn from the account. The interest rate for the first five years was 11% p.a. compounded monthly. Thereafter the interest rate changed to 12% p.a. compounded half-yearly.

5.9.1 Calculate, in terms of x, how much money was in the account 3 years after the initial deposit was made. (This answer should not include the second deposit.) (2)

5.9.2 If the investment was worth R90 132,56 after 8 years, calculate the initial amount that was deposited, i.e. the value of x. (5)

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5.10 John invests an amount of money in an account paying interest at a rate of 15% p.a., compounded monthly. Calculate the annual effective interest rate of this investment. (2)

5.11 Tino invests R500 000 in an account earning interest at the rate of 6% p.a., compounded quarterly. Tino decides to withdraw R11 250 at the end of every 3 months. Tino will continue making these regular withdrawals until there is no money in the account. How many withdrawals of R11 250 will Tino be able to make? (5)

5.12 On **1 March 2021**, Abby made a once-off deposit of R12 000 into an account earning interest at a rate of 9,5% p.a., compounded monthly. She deposited R500 into the same account on **1 April 2023** and continues to make these monthly deposits of R500 on the first day of each month thereafter.

Calculate how much money was in the account immediately after the deposit of R500 is made on **1 March 2025**, exactly four full years after her initial deposit.

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5.13 Six years ago, Thabo bought a phone for R13 000. The value of the phone depreciated annually according to the reducing-balance method. The value of the phone is now R8 337, 75. Calculate the annual rate of depreciation.

(3)

5.14 Eric and Thandi need to save R80 00 each to go on a holiday at the end of December 2027.

- Thandi decides that she will start saving at the end of January 2025. She will make 36 monthly deposits into a savings account that pays interest at 8,6% p.a., compounded monthly. The deposit will be made at the end of each month.
- Eric calculates that if he makes 48 deposits of R1 402, 31 starting at the end of January 2024, he will have enough money to go on holiday. He will make his deposits into a savings account at the end of each month. The savings account pays interest at 8,6% p.a., compounded monthly.

Calculate the difference between the total amount that Eric and Thandi will deposit into their respective accounts over the given period.

(4)

5.15 How many years will it take for an investment to double in value, if it earns interest at a rate of 8,5% p.a., compounded quarterly? (4)

5.16 A company purchased machinery for R500 000. After 5 years, the machinery was sold for R180 000 and new machinery was bought.

5.16.1 Calculate the rate of depreciation of the old machinery over the 5 years, using the reducing-balance method. (4)

5.16.2 The rate of inflation for the cost of the new machinery is 6,3% p.a., over the 5n years. What will the machinery cost at the end of 5 years? (2)

5.16.3 The company set up a sinking fund and made the first payment into this fund on the day the old machinery was bought. The last payment was made three months before the new machinery was purchased at the end of the 5 years. The interest earned on the sinking fund was 10,25% p.a., compounded monthly. The money from the sinking fund and the R180 000 from the sale of the old machinery was used to pay for the new machinery.

Calculate the monthly payment into the sinking fund. (5)

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5.17 Mary's grandparents deposited R5 000 into a savings account on the day that she was born. The account pays interest at a rate of 6,8% p.a., compounded quarterly.
Calculate the accumulated amount in this account on Mary's 16th birthday. (3)

5.18 After 4 years, the value of a printer was half of its original value. Determine the rate at which the value of the printer depreciated over this period, if depreciation was calculated according to a straight-line method. (2)

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5.19 Patrick deposited an amount of R18 500 into an account earning r % interest p.a., compounded monthly. After six months, his balance was R19 319,48.

5.19.1 Calculate the value of r. (3)

5.19.2 Calculate the effective interest rate. (2)

5.20 Kuda bought a laptop for R10 000 on 31 January 2019. He will replace with a new one in 5 years' time on 31 January 2024.

5.20.1 The value of the old laptop depreciates annually at a rate of 20 % p.a. according to the straight-line method. After how many years will the laptop have a value of R0? (2)

5.20.2 Kuda will buy a laptop that costs R20 000. In order to cover the cost price, he made his first monthly deposit into a savings account on 28 February 2019. He will make his 60 monthly deposit on 31 January 2024. The savings account pays interest at 8,7% p.a., compounded monthly. Calculate Kuda's monthly deposit into this account. (4)

5.21 Tino wins a jackpot of R1 600 000. He invests all of his winnings in a fund that earns interest of 11,2% p.a., compounded monthly. He withdraws R20 000 from the fund at the end of each month. His first withdrawal is exactly 1 month after his initial investment.
How many withdrawals of R20 000 will Tino be able to make from this fund? (5)

5.22 A company bought a photocopier for R150 000 on 1 July 2022. They will use the old photocopier as a trade-in when they replace it with a similar new photocopier in 5 years' time on 30 June 2027.

5.22.1 The average rate of inflation over the next 5 years will be 6,5% p.a. Determine the price of a similar new photocopier in 5 years' time. (2)

5.22.2 Calculate the trade-in value of the old photocopier after 5 years, if it depreciates at a rate of 9% p.a., on a straight-line method. (2)

5.22.3 The company set up a sinking fund to cover the replacement cost of the new photocopier. The fund earns interest at the rate of 7,85% p.a., compounded monthly. The company made its first monthly deposit on 31 July 2022 and will continue to do so until 31 May 2027, one month prior to the new photocopier being bought. How much should be deposited at the end of each month so that the company will be able to buy the new photocopier? (4)

DIFFERENTIAL CALCULUS

QUESTION 6

SC/NSC/DBE/2022

6.1 Determine the $f'(x) =$ from the first principles if it is given that $f(x) = -x^2$. (5)

6.2 Determine:

6.2.1 $f'(x)$, if it is given that $f(x) = 4x^3 - 5x^2$ (2)

6.2.2 $D_x\left[\frac{-6\sqrt[3]{x} + 2}{x^4}\right]$ (4)

NSC/DBE/November 2023

6.3 Determine $f'(x)$ from the first princioles if $f(x) = -4x^2$ (5)

6.4 Determine:

6.4.1 $f'(x)$ if $f(x) = 2x^3 - 3x$ (2)

6.4.2 $D_x\left(7.\sqrt[3]{x^2} + 2x^{-5}\right)$ (3)

6.5 For which values of x will the tangent to $f(x) = -2x^3 + 8x$ have a positive gradient? (3)

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6.6 Determine:

6.6.1 $\frac{d}{dx}[3x - 5x^2]$ (2)

6.6.2 g' if $g(x) = \frac{2}{x^2} - \sqrt[3]{x^7}$ (4)

6.7 Given: $f(x) = -6x^2$

6.7.1 Determine $f'(x)$ from the first principles. (5)

6.7.2 Write down how you will restrict the domain of f such that f', the inverse of f, is a function. (1)

6.7.3 Determine the equation of $f^{-1}(x) \leq 0$. Write your answer in the form $y = \ldots$ (3)

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6.8 Determine $f'(x)$ from the first principles if it is given that $f(x) = x^2 - 2$ (5)

6.9 Determine:

6.9.1 $\frac{d}{dx}[3x^2 - 4x]$ (2)

6.9.2 $g'(x)$ if $g(x) = -2\sqrt{x}(x-1)^2$ (4)

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6.10 Determine $f'(x)$ from the first principles if $f(x) = x^2 + x$. (5)

6.11 Determine $f'(x)$ if $f(x) = 2x^5 - 3x^4 + 8x$. (3)

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6.12 Determine $f'(x)$ from the first principles if $f(x) = -2x^2 - 1$. (3)

6.13 Determine:

6.13.1 $f'(x)$, if it is given that $f(x) = -2x^3 + 3x^2$ (2)

6.13.2 $\frac{dy}{dx}$ *if* $y = 2x + \frac{1}{\sqrt{4x}}$ (4)

6.14 The graph $y = f'(x)$ has a minimum turning point at $(1\,;\,-3)$.
Determine the values of x for which f is concave down. (2)

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6.15 Determine $f'(x)$ from the first principles if $f(x) = -2x^2 + x$. (5)

6.16 Determine:

6.16.1 $D_x\left[\frac{-5x}{\sqrt{x}} - \frac{x^2}{5}\right]$ (3)

6.16.2 $\frac{d}{dx}\left[\left(x + \frac{2}{x}\right)\left(x - \frac{2}{x}\right)\right]$ (4)

NSC/DBE/November 2022

6.17 Determine $f'(x)$ from the first principles if $f(x) = x^2 + x$. (5)

6.18 Determine $f'(x)$ if $f(x) = 2x^5 - 3x^4 + 8x$. (3)

6.19 The tangent to $g(x) = ax^3 + 3x^2 + bx + c$ has a minimum gradient at the point $(-1\ ;\ -7)$. For which values of x will g be concave up? (4)

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6.20 Determine $f'(x)$, from the first principles, if $f(x) = \frac{1}{2}x^2$. (4)

6.21 Determine:

6.21.1 $f'(x)$, if $f(x) = \frac{1}{5}x^5 - 6x^{-2}$ (2)

6.21.2 $\frac{d}{dx}\left(x + \sqrt{x}\right)^2$ (4)

SC/May/June 2024

6.22 Determine $f'(x)$ from the first principles if $f(x) = \frac{1}{x}$. (5)

6.23 Determine:

6.23.1 $\frac{d}{dx}\left(\sqrt{4x^6} + \sqrt{2}.x^2\right)$ (3)

QUESTION 7

SC/May/June 2023

Given: $f(x) = x^3 + 4x^2 - 7x - 10$

7.1 Write down the y −intercept of f (1)

7.2 Show that 2 is a root of the equation $f(x) = 0$. (2)

7.3 Hence, factorise $f(x)$ completely. (3)

7.4 If it is further given that the coordinates of the turning points are approximately at $(0{,}7\ ;\ -12{,}6)$ and $(-3{,}4\ ;\ 20{,}8)$, draw a sketch of f and label all intercepts and turning points. (3)

7.5 Use your graph to determine the values of (x) for which:

7.5.1 $f'(x) < 0$ (2)

7.5.2 The gradient of a tangent to f will be a minimum (2)

7.5.3 $f'(x) \cdot f''(x) \leq 0$. (3)

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7.6 Given: $f(x) = -x^3 + 12x - 16$

7.6.1 Show that $(x - 2)$ is a factor of $f(x)$. (2)

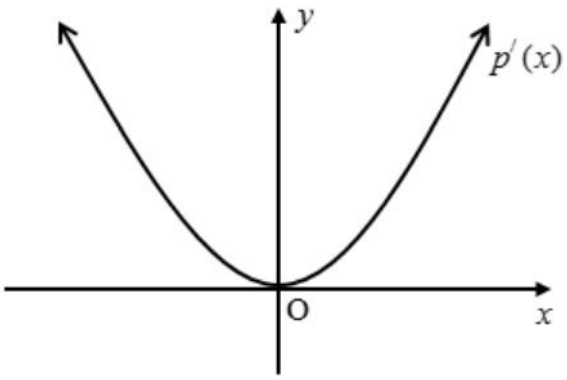
7.6..2 Determine the $x-$ intercepts of f. (3)

7.6.3 Determine the coordinates of the turning points of f. (4)

7.6.4 Sketch the graph of f, clearly indicating turning points and intercepts with the axes (3)

7.6.5 Determine the equation of the tangent at the point of inflection. (4)

7.7 A sketch graph of $p'(x)$ is given below.



7.7.1 For which values of (x) is the graph of $p(x)$ increasing? (2)

7.7.2 For which values of (x) is the graph of $p(x)$ concave up? (2)

SC/DBE/May/June 2025

Given: $f(x) = x^3 - 6x^2 + 9x - 4 = (x - 4)(x - k)^2$

7.8 Show that $k = 1$. (2)

7.9 Calculate the coordinates of the turning point of f. (4)

7.10 Describe the concavity of f at $x = -3$ (2)

7.11 Draw the graph of f. Label ALL turning points and intercepts with the axes. (4)

Given: $f(x) = -x^3 + 6x^2 - 9x + 4 = (x-1)(-x+4)$

7.12 Determine the coordinates of the turning points of f. (4)

7.13 Draw the graph of f. Clearly label all the intercepts with the axes and any turning points. (4)

7.14 Use the graph to determine the value(s) of k for which $-x^3 + 6c^2 - 9x + 4 = k$ will have three real and unequal roots.

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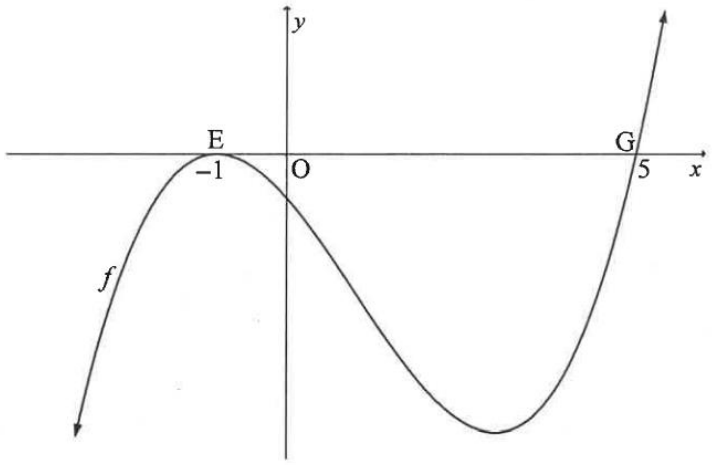
7.15 The graph of $f(x) = ax^3 + bx^2 + cx - 5$ is drawn below. E$(-1\,;0)$ and G$(5\,;0)$ are the $x-$ intercepts of f.



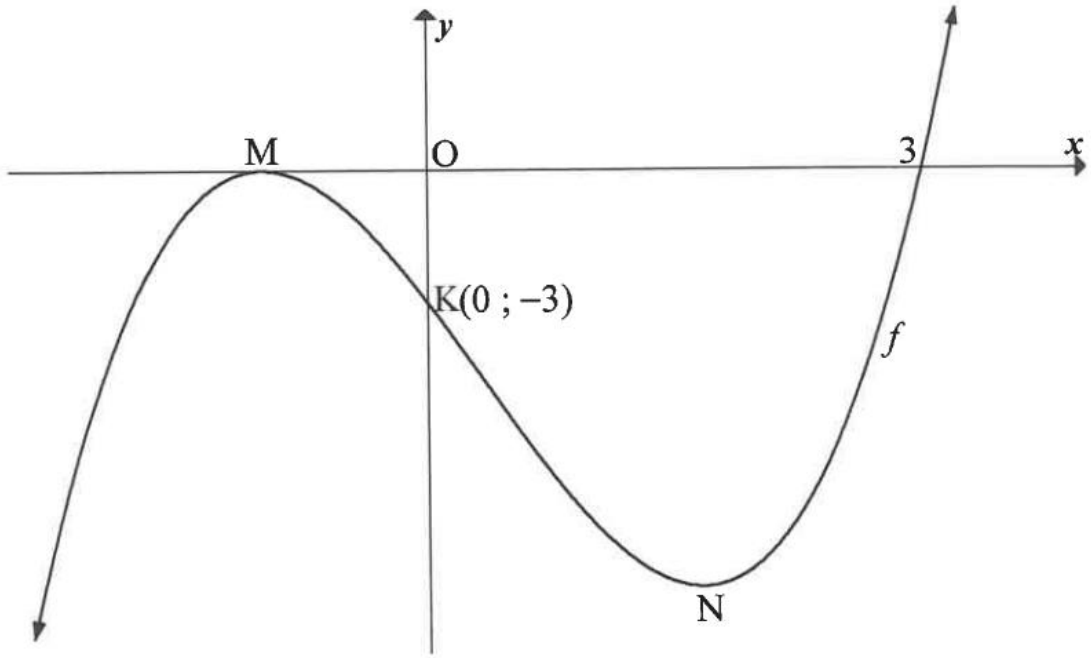
7.15.1 Show that $a = 1$, $b = -3$ and $c = -9$. (3)

7.15.2 Calculate the value of x for which f has a local minimum value. (3)

7.15.3 Use the graph to determine the value of x for which $f''(x).f(x) > 0$. (3)

7.16 Sketched below is the graph of $f(x) = x^3 + ax^2 + bx + c$.
The x −intercepts of f are (3 ; 0) and M, where M lies on the negatives x −axis.

K$(0\ ;\ -3)$ is the $y-intercept$ of f.
M and N are the turning points of f.



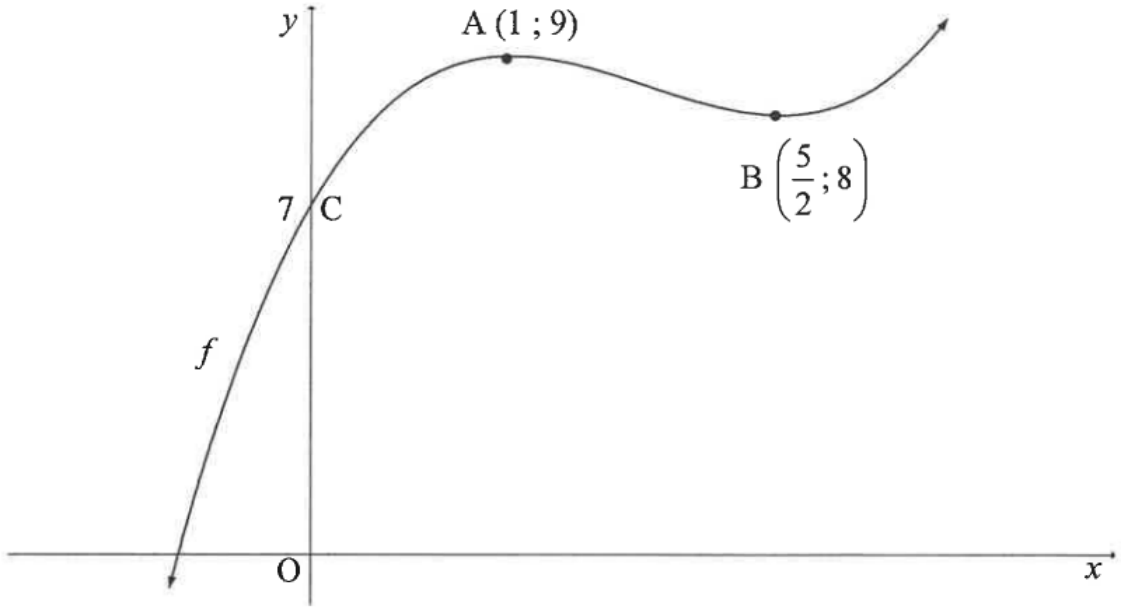
7.16.1 Show that the equation of f is given by $f(x) = x^3 - x^2 - 5x - 3$. (5)

7.16.2 Calculate the coordinates of N. (5)

7.16.3 For which values of x will:

(a) $f(x) < 0$ (2)
(b) f is increasing (2)
(c) f be concave up (3)

7.17 A(1 ; 9) and B$\left(\frac{5}{2}\,;8\right)$ are the turning points of graph f below.
C $(0\,;7)$ is the $y-intercept$ of f.



7.17.1 For which values of x if f decreasing? (2)

7.17.2 Write down the x – intercept of f' , the derivative of f. (2)

7.17.3 For which values of x will f be concave up? (2)

7.17.4 Determine the value of k for which $y = f(x) + k$ will have THREE positive x–intercepts. (2)

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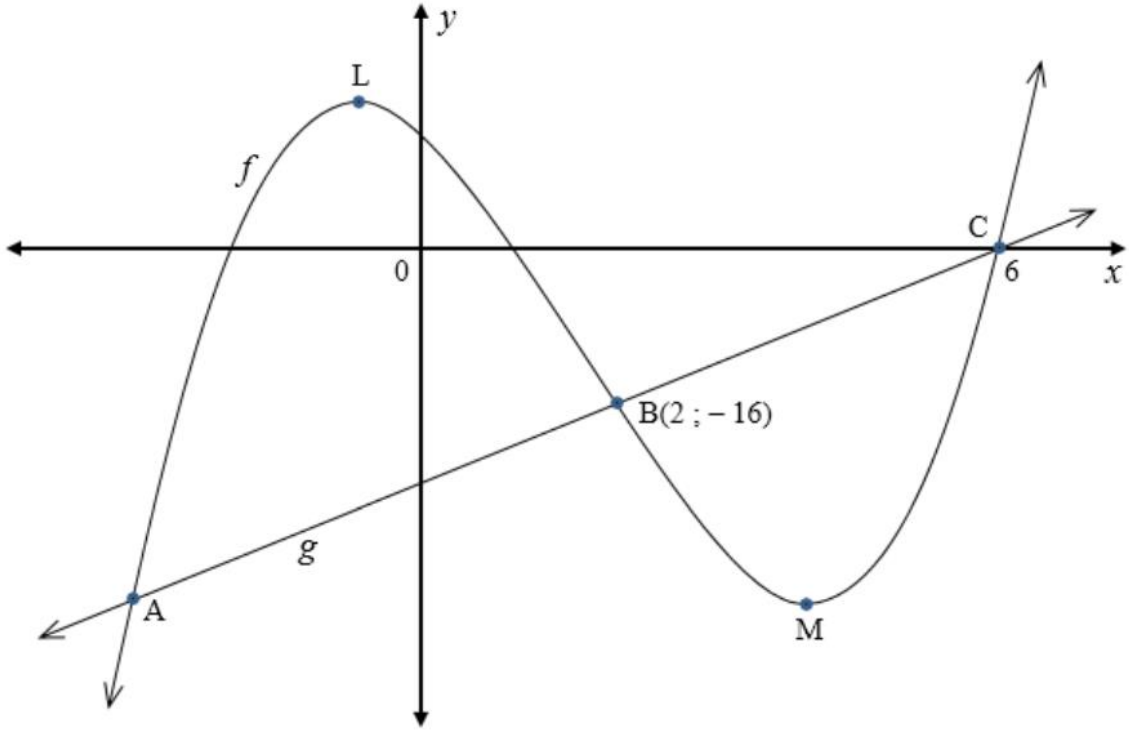
7.18 The sketch below represents the functions of $f(x)\ x^3 + bx^2 + cx + d$ and $g(x) = ax + q$.
The points A , B $(2\ ;\ -16)$ and C are the points where two graphs intersect.
C(6 ; 0) is the x −intercept of f , while L and M are the turning points of f.



7.18.1 Show that $b = -5$, $c = -8$ and $d = 12$ if it given that, $f'(x) = 3x^2 - 10x - 8$. (4)

7.18.2 Determine the coordinates of the turning points, L and M, of f. (5)

7.18.3 Determine the equation of g. (3)

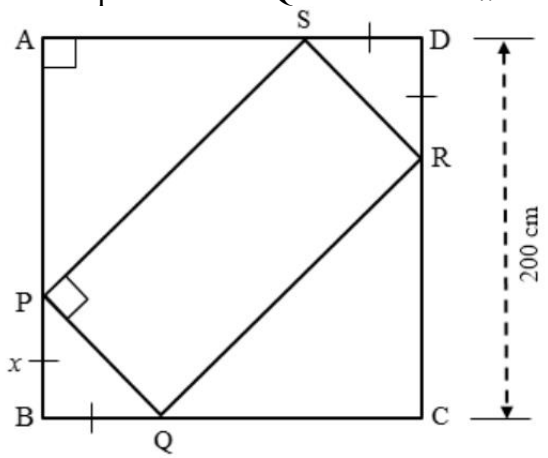
7.18.4 If it is further given that the coordinates of point A are $(x\ ;\ -36)$, determine the length of AM. (3)

7.18.5 For which value(s) of x:

(i) is the graph, f increasing? (2)

(ii) Is the graph, f concave down? (2)

8 In the diagram below, ABCD is a square with side length CD = 200 cm. PQRS is a rectangle with vertices on the sides of the square. PB = BQ = SD = DR = x cm



8.1 Show that the area of the rectangle is given by, $A = 2(200x - x^2)$. (3)

8.2 Determine the value of x for which the area of the rectangle will be a maximum (3)

8.3 What is the ratio of the maximum area of PQRS: area of ABCD? (3)

PROBABILITY

QUESTION 9

NSC/DBE/November 2024

9.1 A certain number of learners are sitting for examinations in Mathematics, Tourism and Geography.

- All these learners sit for at least one of these examinations.
- The total number of learners who sit for Mathematics (M), is 22.
- The total number of learners sitting for Tourism (T), is 16.
- The total number of learners sitting for Geography (G), is 18.
- 5 learners sit for Mathematics and Tourism, but not Geography.
- 4 learners sit for Mathematics and Geography, but not Tourism.
- 3 learners sit for Tourism and Geography, but not Mathematics.
- 6 learners sit for only Tourism.

9.1.1 Draw a Venn diagram to present ALL the learners sitting for these examinations. (3)

9.1.2 Calculate the probability that a learner, chosen at random, will sit for examinations in at least TWO of the subjects. (2)

9.1.3 Determine if the events: sitting for examinations in Mathematics and sitting for examinations in Tourism are independent. Support your answer with necessary calculations. (4)

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9.2 A group of people participated in a trial to test a new headache pill.

- 50% of the participants received the headache pill.
- 50% of the participants received a sugar pill.
- $\frac{2}{5}$ of the group receiving the headache pill were not cured.
- $\frac{3}{10}$ of the group receiving the sugar pill were cured.

9.2.1 Represent the given information on a tree diagram. Indicate on your diagram the probability associated with each branch as well as the outcomes. (3)

9.2.2 Determine the probability that a person chosen at random from the group will NOT be cured.

9.3 Three events, A, B, and C, are considered:

$P(A) = \frac{2}{5}$, $P(B) = \frac{1}{4}$ and $P(A \text{ or } B) = \frac{13}{20}$.

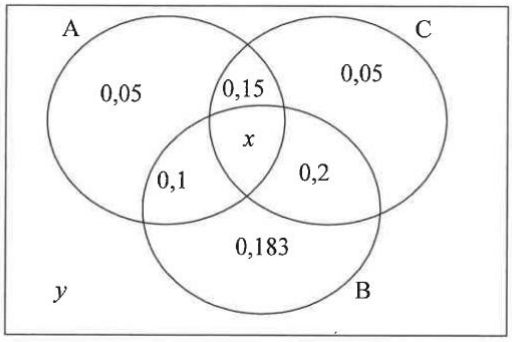
9.3.1 Are the events A and B mutually exclusive? Support your answer with the necessary calculations. (2)

9.3.2 Determine P (only C), if it is further given that

$P(A \text{ or } C) = \frac{7}{10}$, $P(A \text{ or } C) = \frac{2}{5}$, and $2P(B \text{ and } C) = P(A \text{ and } C)$. (3)

9.3.3 Determine the probability that events A, B, or C do NOT take place. (2)

9.4 A, B, and C ARE THREE EVENTS. The probabilities of these events (or any combination of them) occurring is given in the Venn-diagram below.



9.4.1 If it is given that the probability that at least one of the events will occur is 0,893. Calculate the value of:

(a) *y,* the probability that none of the events will occur. (1)

(b) *x,* the probability that all three events will occur. (1)

9.4.2 Determine the probability that at least two events will take place. (2)

9.4.3 Are events B and C independent? Justify your answer (5)

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9.5 A and B are mutually exclusive events.
If $P(A) = 0{,}42$ and $P(A \text{ or } B) = 0{,}79$. Calculate: P(B) (2)

EC/June 2022

9.6 In a survey, 1 530 people were asked if they had ever broken a limb. The results of the survey were as follows:

	Broken a limb	**Not Broken a limb**	**Total**
Male	463	b	782
Female	*a*	*c*	*d*
Total	913	617	1 530

9.6.1 Calculate the values of *a, b, c,* and *d*. (4)

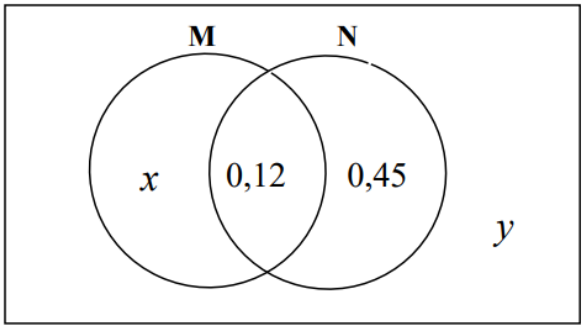
9.6.2 If a person is chosen at random, what is the probability that it will be a female who has not broken a limb? (2)

9.7 Two learners are selected at random from group of 10 boys and 12 girls. Determine the probability that:

9.7.1 They are both girls (2)

9.7.2 One is a boy and one is a girl (3)

9.8 In the Venn diagram below, M and N are independent events.



Calculate, giving answers correct to two decimals places:

9.8.1 The value of x. (3)

9.8.2 The value of y. (2)

9.9 ***EC/June 2024***

Events A and B are independent events. It is further given that:

- P(A) = 0,6
- P(B) = 0,5

9.9.1 Are the events mutually exclusive? Motivate your answer (2)

9.9.2 Represent the information on a Venn-diagram (3)

9.9.3 Calculate:

(a) P (only A) (1)

(b) P(not A or not B) (2)

9.10 The contingency table below represents 100 learners' responses regarding camping

	Boys	Girls	Total
Like Camping	24	30	54
Dislike Camping	14	32	46
Total	38	62	100

9.10.1 If a learner from this group is chosen randomly, what is the probability that it is a girl? (1)

9.10.2 Is the event, "like camping" independent of the gender? (4)

9.11 There are only red balls and green balls in a bag. A ball is taken at random from the bag. The probability that the ball is green is $\frac{3}{7}$. The ball is replaced in the bag. 2 more red balls and 3 more green balls are put in the bag. Thereafter, a ball is taken at random from the bag and the probability that this ball is green is $\frac{6}{13}$.

Determine how many of each colour ball was originally in the bag. (5)

NSC/DBE/November 2024

10.1 A company generates a 4-character code using the 26 letters of the alphabet and the 10 digits, from 0 to 9.

The code is in the form:

letter	Digit	Letter	digit

10.1.1 Determine how many different codes can be formed if letters and digits may be repeated. (2)

10.1.2 Determine how many different codes can be formed if:

- The letters D, F, I, Q, U, and V, may not be used
- The code may NOT start with a W or a Z
- Letters or digits may NOT be repeated
- The code ends with an odd digit (4)

10.1.3 The company wishes to increase the number of 4-character codes formed in QUESTION 10.1.2 by allowing letters D, F, I, Q, U, and V to be used.
Calculate the percentage increase in the number of different codes that can now be formed. (2)

NSC/DBE/November 2022

10.2 A four-digit code is required to open a combination lock. The code must be even-numbered and may not contain the digits 0 or 1. Digits may not be repeated.

10.2.1 How many possible 4-digit combinations are there to open the lock? (3)

10.2.2 Calculate the probability that you will open the lock at the first attempt if it is given that the code is greater than 5 000 and the third digit is 2. (5)

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10.3 Seven friends (4 boys and 3 girls) want to stand in a straight line next to each other to take a photo.

10.3.1 In how many ways can the 3 girls stand next to each other in the photo? (2)

10.3.2 In the next photo, determine the probability that Selwyn (a boy) and Lindiwe (a girl)will NOT stand to each other in the photo. (3)

10.4 Consider the three-digit numbers from 501 up to 999

10.4.1 How many of these three-digit numbers have exactly one 5 in them (4)

10.4.2 Calculate the probability of a three-digit number not satisfying the condition given in QUESTION 10.4.1 (3)

ANALYTICAL GEOMETRY GRADE 10 – 12 NOTES

This topic is always split into two questions:

1. **A question dealing with lines, quadrilaterals and triangles**
2. **A question dealing with lines and circles.**

Analytical Geometry: application of straight line functions in conjunction with Euclidean Geometry by using points on a Cartesian Plan.

Straight line parallel to the x-axis: m = 0

Straight line parallel to the y-axis: m = undefined

Straight line equation: $y = mx + c$ or $y - y_1 = m(x - x_1)$

Note: Always look for gradient and one point

- ❖ Find the equation of a medium, altitude, or perpendicular bisector.
- ❖ Find the equation of a tangent.
- ❖ Determine whether a point is on a graph or not
- ❖ Find points of intersection of graphs.

Gradient formula: $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

Use to:

- Determine whether three points are collinear (in a straight line)

Co – linear points: $m_{AB} = m_{BC} OR$
$d_{AB} + d_{BC} = d_{AC}$

Collinear points A, B and C lie on the same line.

- Determine Parallel ($\|$) lines: $m_1 = m_2$
- Perpendicular ($\perp$) lines : $m_1 \times m_2 = -1$
- Determine **angle** of inclination: $\tan\theta = m$
  $\Rightarrow \theta$ is the angle of inclination($\angle$ between line and positive x – axis) and angle of inclination is also used to determine the angle between two lines(or angles in Δ).
- Determine whether a quadrilateral is a parallelogram
- Show that a triangle is **right – angled.**

Distance : $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ OR $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

Use to:

- ➢ Find the distance between two points
- ➢ Prove that a triangle is isosceles /equilateral/ right – angled .
- ➢ Prove that a quadrilateral is a parallelogram

- Determine perimeter and area

Midpoint formula: $M(x;y) = (\frac{x_2 + x_1}{2}; \frac{y_2 + y_1}{2})$

Use to:

- Find the midpoint of a line segment

1. **Find the point of intersection of two lines**

- The point of intersection is where the two lines are equal to each other, **you have to make the two equations equal to each other.**
- Solve for x
- Substitute the x-coordinate found into either of the two equations to get the y – coordinate of the point.

2. **(a) Find the equation of a median of a triangle**

A **median** of a triangle is a line from one vertex of the triangle to the midpoint of the opposite side.

When you have the coordinates of the three vertices of a triangle , you can find the equation of the median by:

- Determine **which side** the median is bisecting, it is bisecting. It will always bisect the side OPPOSITE to the given vertex.
- Calculate the **midpoint** of the side it is bisecting, using the midpoint formula.
- Now calculate the **gradient** of the median, using the coordinates of the vertex given and the midpoint calculated in step 2.
- **Substitute** either the vertex given or the midpoint calculated in step 2 into the straight line equation and simplify.

(b) Find the equation of an altitude/ height of a triangle

An **altitude** of a triangle is a line from one vertex of a triangle, **perpendicular** to the side opposite that vertex.

When you have the coordinates of the three vertices of a triangle, you can find the equation of the altitude by:

I. Determine **which side** the altitude is perpendicular to. It will always be perpendicular to the side OPPOSITE the given vertex
II. Calculate the **gradient** of this side, using the coordinates of the two vertices given.
III. Now calculate the **gradient** of the altitude, using the fact that if two lines are perpendicular, the **product of their gradients is equal to -1.**
IV. **Substitute** the vertex given into the straight line equation and simplify.

(c) Find the equation of a perpendicular bisector of a line

A perpendicular bisector of a line is a line that is **perpendicular** to another line and also **bisects** the other line (goes through the midpoint of that line).

When you have the coordinates of the three vertices of a triangle, you can find the equation of the perpendicular bisector by:

i. Determine **which side** the line is perpendicular to. This will be stated in the question.
ii. Calculate the **gradient** of the side it is perpendicular to, using the coordinates of the two vertices given.
iii. Now calculate the gradient of the perpendicular bisector, using the fact that if two lines are perpendicular, the **product of their gradients is equal to -1.**
iv. Calculate the **midpoint** of the side that the perpendicular bisector is perpendicular to.
v. Now substitute both the **gradient** of the perpendicular bisector , as well as the **midpoint** found in iv, into the straight line equation and simplify.

3. Find the coordinates of the fourth vertex of a parallelogram

We find the coordinates of the fourth vertex of a parallelogram **by inspection.** Pay special attention to the **Name** of the parallelogram as stated in the question. **The order of the vertices in the name is the same as the order of the vertices of the parallelogram.**

For example, if the name is **PQRS,** that means that P is connected to Q, Q is connected to R, R is connected to S and S is the connected back to P.

In order to find the coordinates of the fourth vertex, first **determine where the vertex will be** by looking at the name of the parallelogram. Then look at the **change in coordinates from one vertex to another and** apply that change in coordinates in order to calculate the coordinates of the fourth vertex.

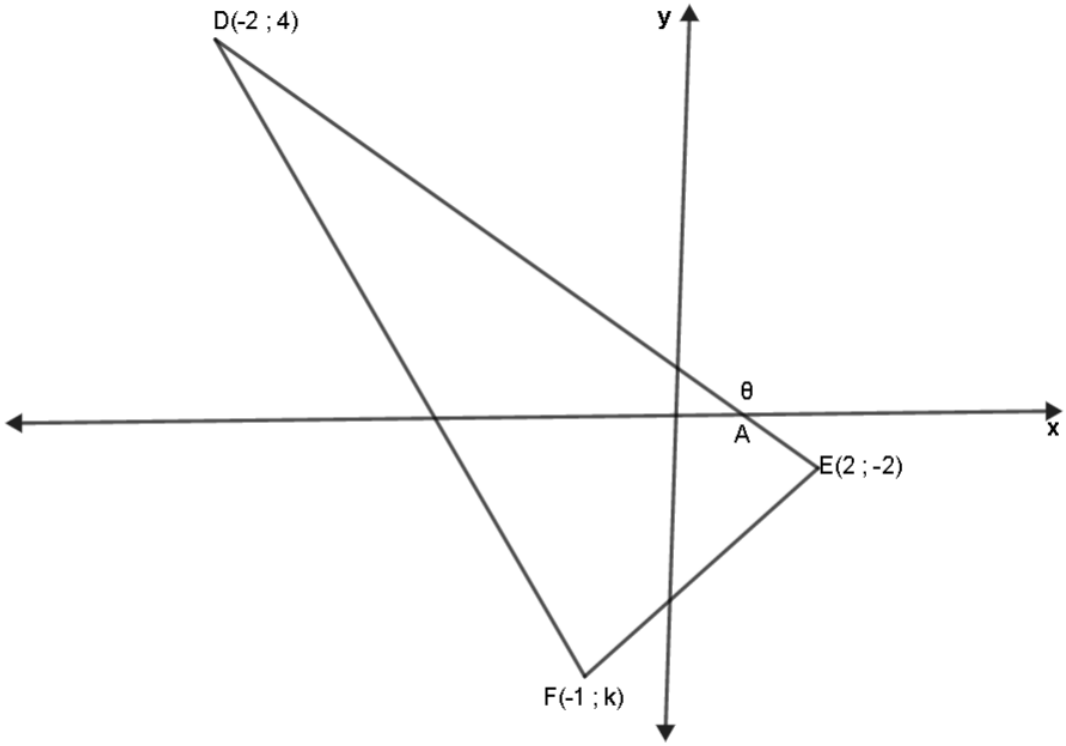
4. Calculate the area of a figure

You will often be asked to calculate the area of a figure that consists of a right – angled triangle and one that is not a right – angled.

Worked Examples

Example 1 adapted from Star Schools Mathematics Workbook Grade 12

In the diagram, D(-2;4), E(2;-2) and F(-1; k) are three points in a Cartesian plane, $\hat{E} = 90°$; $D\hat{A}x = \theta$.



(a) Calculate the gradient of DE

(b) Calculate the size of $D\hat{A}x = \theta$ round of to two decimal digits.

(c) Show that k = -4

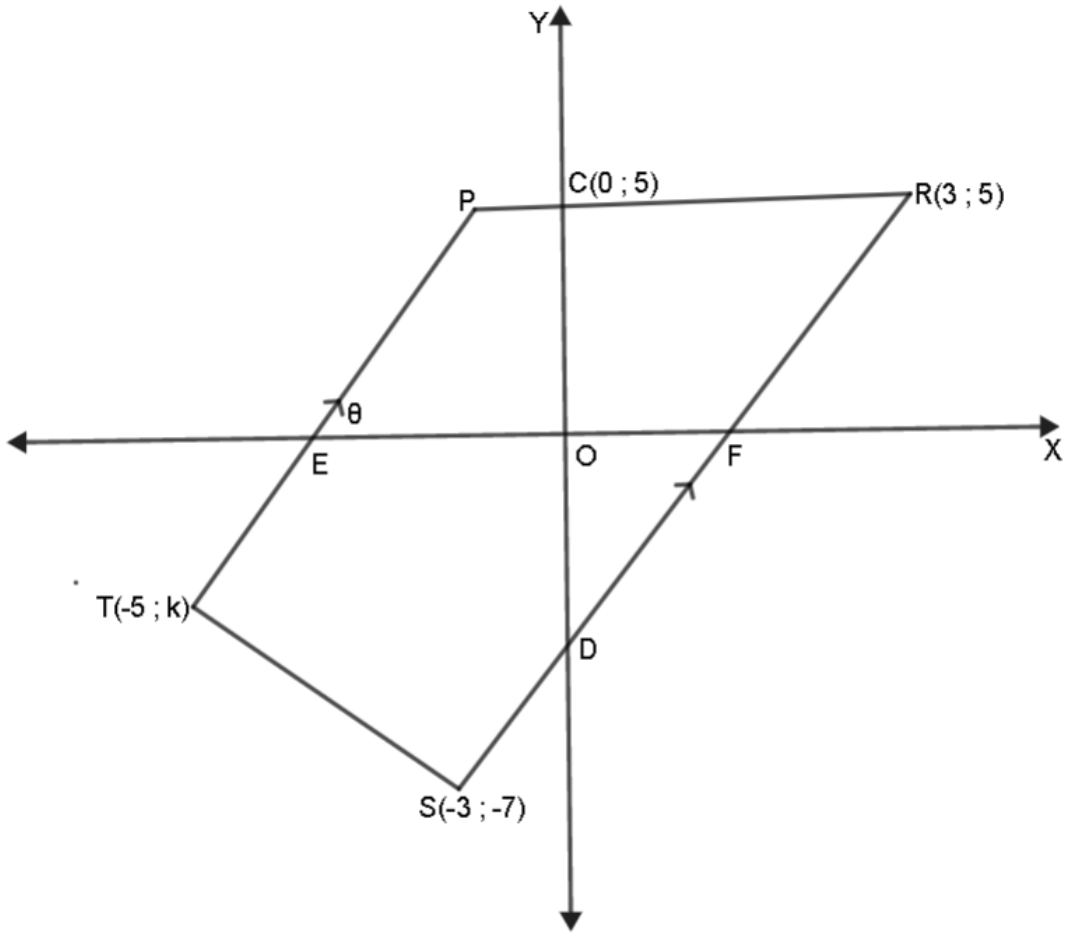
(d) Hence calculate the coordinates of M, the midpoint of FE.

(e) Determine the equation of the straight line parallel to DE, which passes through M.

(f) Calculate the area of Δ DEF.

Example 2 adapted from Mathematics Nov 2019

In the diagram, P, R(3;5), S(-3 ; -7) and T(-5 ; k) are vertices of trapezium PRST and PT || RS. RS and PR cut the y-axis at D and C(0 ; 5) respectively. PT and RS cut the x – axis at E and F respectively. $P\hat{E}F = \theta$.



2.1. Write down the equation of PR.

2.2. Calculate the:

2.2.1. Gradient of RS.

2.2.2. Size of θ

2.2.3 Coordinates of D

2.3. If it is given that TS = $2\sqrt{5}$, calculate the value of k.

2.4. Parallelogram TDNS, with N in the 4th quadrant, is drawn. Calculate the coordinates of N.

EQUATION OF A CIRCLE AND TANGENT TO CIRCLE

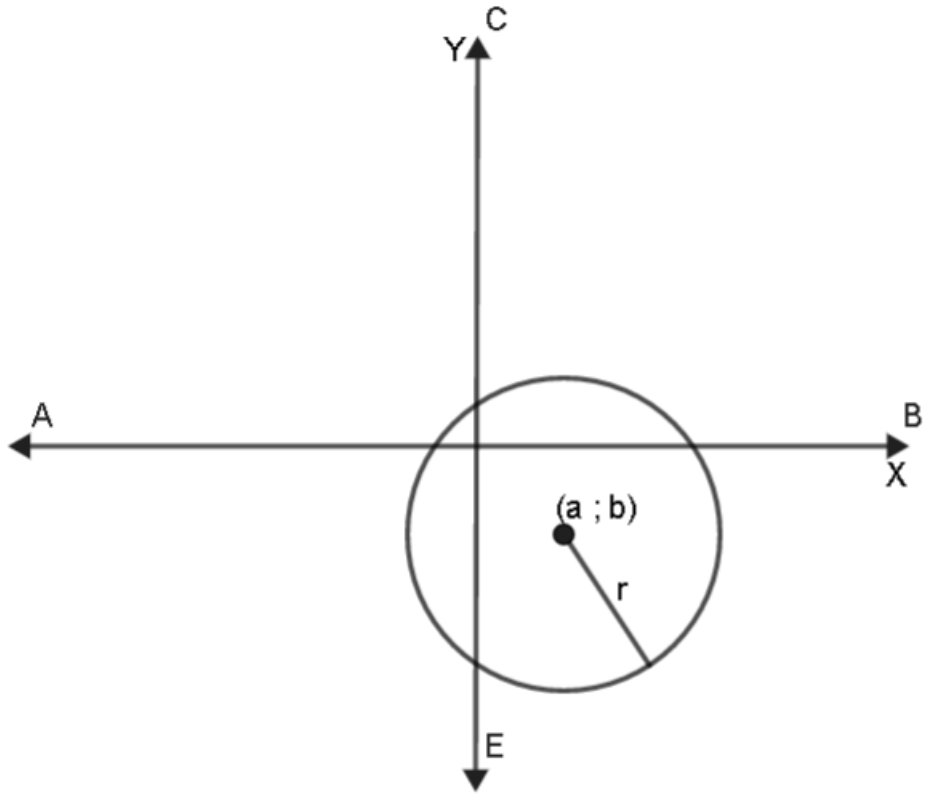
Prior Knowledge:

i. All analytical formulae (distance, midpoint, gradients, and straight line functions)
ii. Euclidean Geometry (types of triangles and quadrilaterals, circle geometry theorems)

The equation of a circle is given by the equation:

$$(x-a)^2+(y-b)^2=r^2,$$

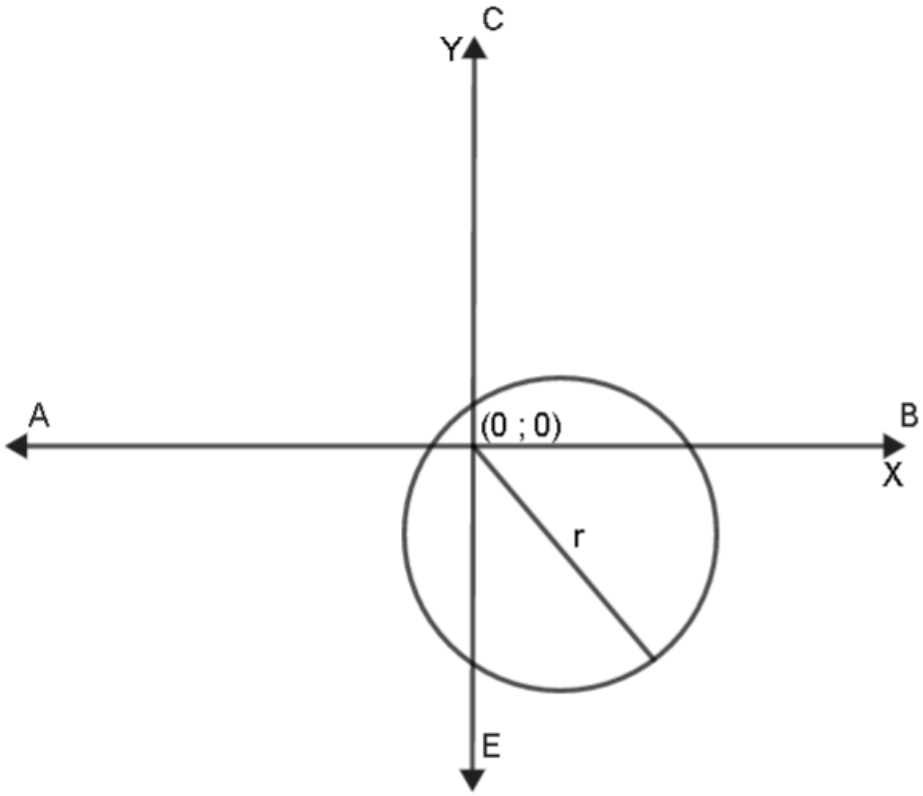
where (a ; b) is the centre of the circle, ad x and y are coordinates of a point on the circle.



With radius as the subject:

$r = \sqrt{(x-a)^2+(y-b)^2}$

If the centre of the circle is at the origin (0 ; 0)



$$x^2 + y^2 = r^2$$

$$r = \sqrt{x^2 + y^2}$$

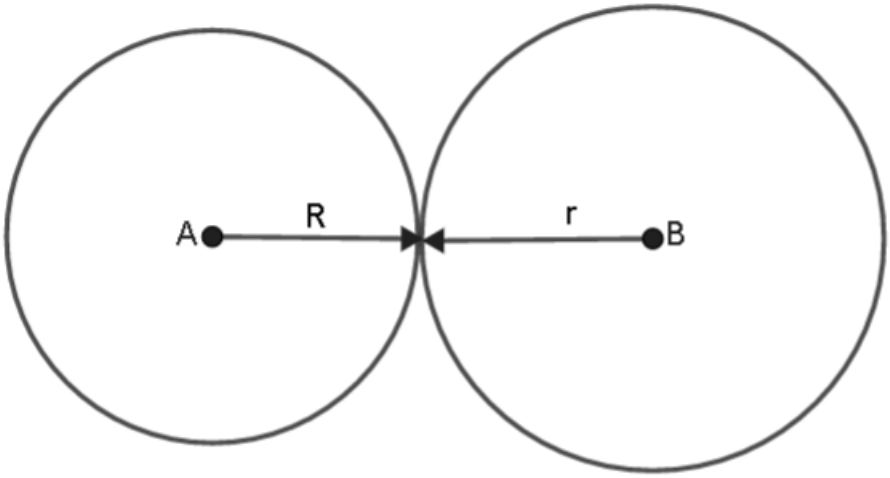
The standard centre – radius equation can be expanded to give;

$Ax^2 + Bx + Cy^2 + Dy + E = 0$

Given two circles with centres A and B respectively,

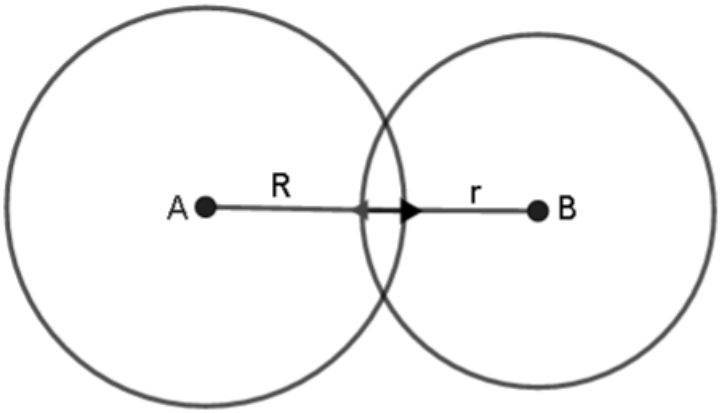
It can be determined if the circles are:

(a) Touching externally (one point of intersection)



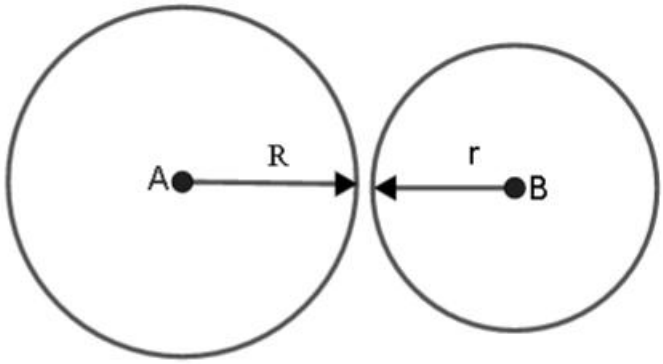
AB = R+ r

(b) Touching internally (Two points of intersection)



AB < R + r

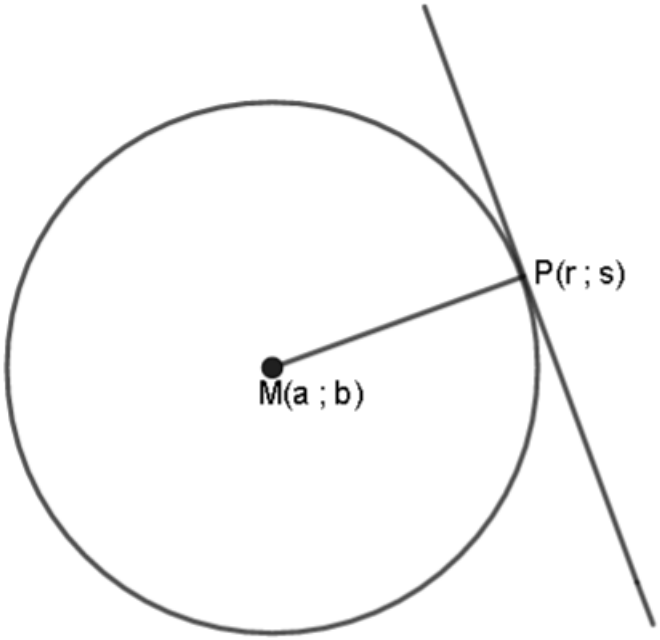
(c) Not touching at all



AB > R + r

TANGENTS TO CIRCLES

- A line that touches a circle at one point only is called a tangent, i.e. there is a single point of intersection.
- The radius of a circle and the tangent to the circle are perpendicular to each other.
- Two tangents coming from the same point of contact are equal in length.



To determine the equation of a tangent to a circle $(x - a)^2 + (y - b)^2 = r^2$ at a point P(a ; b), follow the steps below:

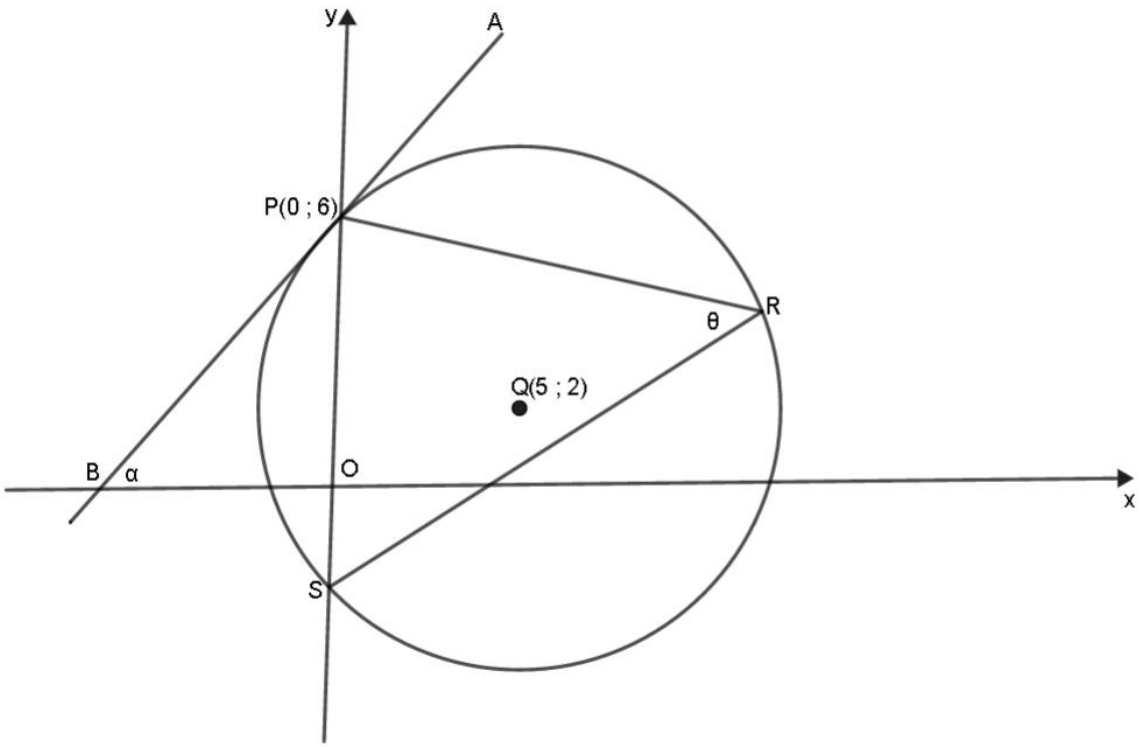
- Find the gradient of the radius MP. MP is the line through (a ; b) and (r ; s).
- Determine the gradient of the tangent using the fact that the tangent and the radius are perpendicular to each other.
- Use the equation of a straight line and substitute on the tangent gradient and the point of contact to find the equation of the tangent.

Worked Examples

Example 1 adapted from Mathematics November Paper 2 2015

1. In the diagram below, Q(5 ; 2) is the centre of a circle that intersects the y- axis at P(0 ; 6) and S. The tangent APB at P intersects the x – axis at B and makes the angle α with the positive

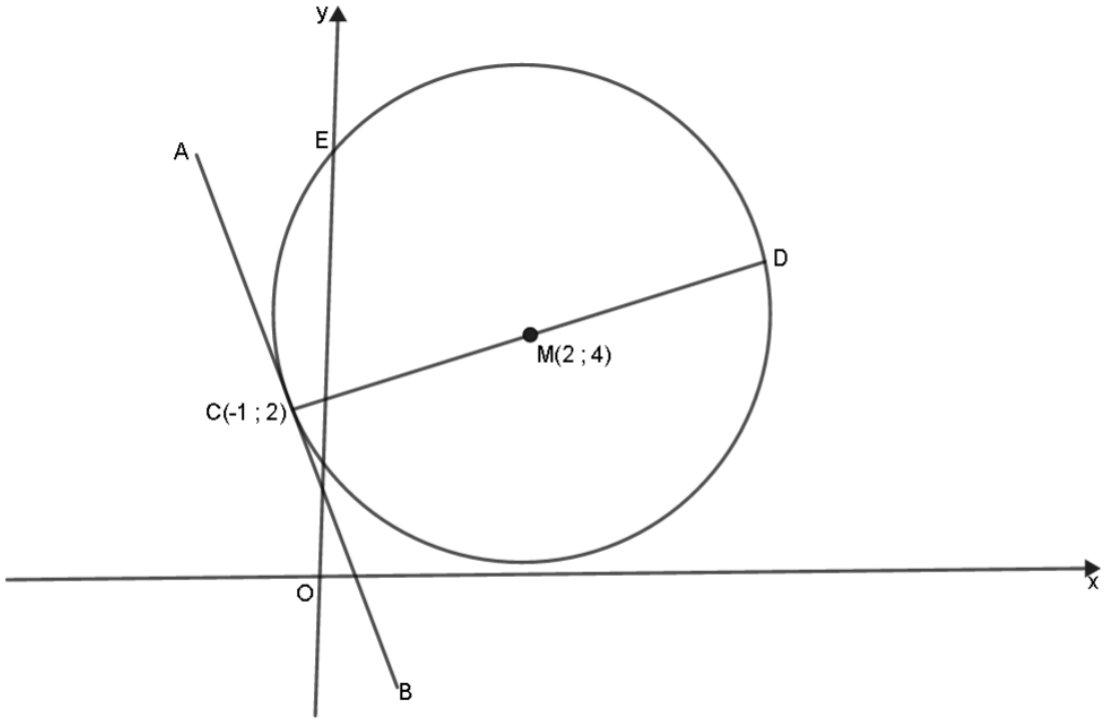
x – axis. R is a point on the circle and $P\hat{R}S = \theta$.



1.1 Determine the equation of the circle in the form $(x - a)^2 + (y - b)^2 = r^2$.
1.2 Calculate the coordinates of S.
1.3 Determine the equation of the tangent APB in the form y = mx + c.
1.4 Calculate the size of α.
1.5 Calculate, with reasons, the size of θ.
1.6 Calculate the area of ΔPQS.

Example 2 adapted from Mathematics February Paper 2 2015

In the diagram below, the circle centred at M(2 ; 4) passes through C(-1 ; 2) and cuts the y – axes at E. The diameter CMD is drawn and ACB is a tangent to the circle.



2.1. Determine the equation of the circle in the form $(x - a)^2 + (y - b)^2 = r^2$.
2.2. Write down the coordinates of D
2.3. Determine the equation of AB in the form $y = mx + c$.
2.4. Calculate the coordinates of E.
2.5. Show that EM is parallel to AB.

3. Determine whether or not the circles having equations $(x + 2)^2 + (y - 4)^2 = 25$ and $(x - 5)^2 + (y + 1)^2 = 9$ will intersect. Show **all** calculations.

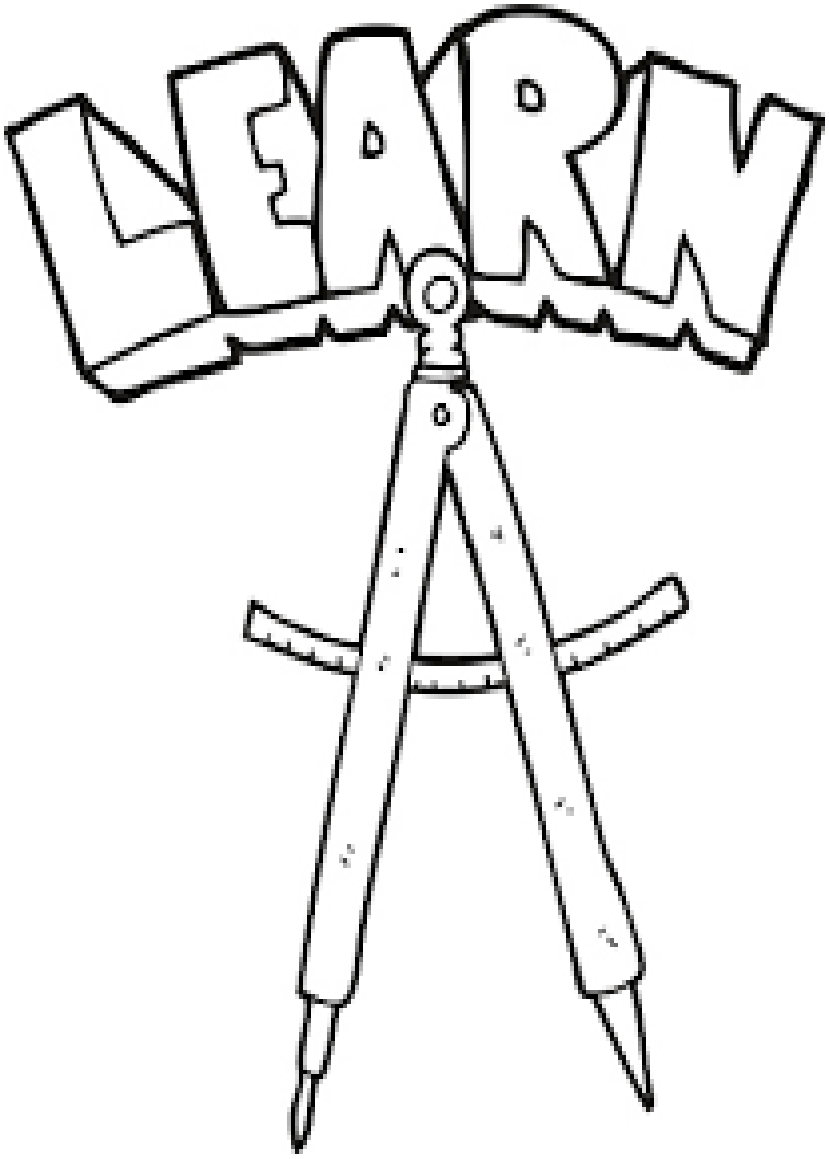
basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

MATHEMATICS P2 TOPICS
GRADE 12

QUESTIONS ORGANISED BY TOPICS
DBE EXAMS FROM MARCH 2018 – JUNE 2024



MATHEMATICS

DBE/March -2018

QUESTION 1

An organisation decided that it would set up blood donor clinics at various colleges. Students would donate blood over a period of 10 days. The number of units of blood donated per day by students of college X is shown in the table below.

DAYS	1	2	3	4	5	6	7	8	9	10
UNITS OF BLOOD	45	59	65	73	79	82	91	99	101	106

1.1 Calculate:

1.1.1 The mean of the units of blood donated per day over the period of 10 days (2)

1.1.2 The standard deviation of the data (2)

1.1.3 How many days is the number of units of blood donated at college X outside one standard deviation from the mean? (3)

1.2 The number of units of blood donated by the students of college X is represented in the box and whisker diagram below.

1.2.1 Describe the skewness of the data. (1)

1.2.2 Write down the values of **A** and **B**, the lower quartile and the upper quartile respectively, of the data set. (2)

1.3 It was discovered that there was an error in counting the number of units of blood donated by college X each day. The correct mean of the data is 95 units of blood. How many units of blood were NOT counted over the ten days? (1)

[11]

QUESTION 1

The monthly profit (in thousands of rands) made by a company in a year is given in the table below.

110	112	156	164	167	169
171	176	192	228	278	360

1.1 Calculate the:

1.1.1 Mean profit for the year (3)

1.1.2 Median profit for the year (1)

1.2 On the number line provided in the ANSWER BOOK, draw a box and whisker diagram to represent the data. (2)

1.3 Hence, determine the interquartile range of the data. (1)

1.4 Comment on the skewness in the distribution of the data. (1)

1.5 For the given data:

1.5.1 Calculate the standard deviation (1)

1.5.2 Determine the number of months in which the profit was less than one standard deviation below the mean (2)

[11]

DBE/2021

QUESTION 1

1.1 Sam recorded the amount of data (in MB) that she had used on each of the first 15 days in April. The information is shown in the table below.

26	13	3	18	12	34	24	58	16	10	15	69	20	17	40

1.1.1 Calculate the:

(a) Mean for the data set (2)

(b) Standard deviation for the data set (1)

1.1.2 Determine the number of days on which the amount of data used was greater than one standard deviation above the mean. (2)

1.1.3 Calculate the maximum total amount of data that Sam must use for the remainder of the month if she wishes for the overall mean of April to be 80% of the mean for the first 15 days. (3)

1.2 The wind speed (in km per hour) and temperature (in °C) for a certain town were recorded at 16:00 for a period of 10 days. The information is shown in the table below.

WIND SPEED IN km/h (x)	2	6	15	20	25	17	11	24	13	22
TEMPERATURE IN °C (y)	28	26	22	22	16	20	24	19	26	19

1.2.1 Determine the equation of the least squares regression line for the data. (3)

1.2.2 Predict the temperature at 16:00 if, on a certain day, the wind speed of this town was 9 km per hour. (2)

1.2.3 Interpret the value of b in the context of the data. (1)

[14]

QUESTION 2

It is said that the number of times that a cricket chirps in a minute gives a very good indication of the air temperature (in °C). The table below shows the information recorded during an observation study.

CHIRPS PER MINUTE	AIR TEMPERATURE IN °C
32	8
40	10
52	12
76	15
92	17
112	20
128	25
180	28
184	30
200	35

2.1 Represent the data above on the grid provided in the ANSWER BOOK. (3)

2.2 Explain why the claim, 'gives a very good indication', is TRUE. (1)

2.3 Determine the equation of the least squares regression line of the data. (3)

2.4 Predict the air temperature (in °C) if a cricket chirps 80 times a minute. (2)
[9]

QUESTION 1

The ordered data below lists the distances (in kilometres) travelled daily by a sales representative in a certain month.

131 132 140 141 144 146 147 149 150 151

155 159 163 166 167 169 175 178 187 189

Class interval	Frequency	Cumulative frequency
$130 \leq x < 140$	2	2
$140 \leq x < 150$	6	8
$150 \leq x < 160$	4	12
$160 \leq x < 170$	4	16
$170 \leq x < 180$	2	18
$180 \leq x < 190$	2	20

1.1 Calculate the mean of the given data. (2)

1.2 Calculate the standard deviation of the given data. (2)

1.3 From the cumulative frequency table shown above, draw an Ogive (curve) of the given data on the grid provided. (3)

1.4 Draw the box and whisker diagram for the given data. (3)

1.5 Comment on the skewness of the data. (1)

[11]

QUESTION 2

The number of days that Grade 8 learners were absent at a certain high school during a year was recorded. This information is represented in the table below.

NUMBER OF DAYS ABSENT	NUMBER OF LEARNERS
$0 \leq x < 5$	34
$5 \leq x < 10$	45
$10 \leq x < 15$	98
$15 \leq x < 20$	43
$20 \leq x < 25$	7
$25 \leq x < 30$	3

2.1 Write down the modal class for the data. (1)

2.2 How many learners were absent from school for less than 15 days? (1)

2.3 How many Grade 8 learners are at the school? (1)

2.4 Draw a cumulative frequency graph (ogive) to represent the data above on the grid provided in the ANSWER BOOK. (4)

2.5 Use the cumulative frequency graph to determine the median number of days the Grade 8 learners were absent. (2)

[9]

QUESTION 1

The table below shows the mass (in kg) of the school bags of 80 learners.

MASS (in kg)	FREQUENCY
$5 < m \leq 7$	6
$7 < m \leq 9$	18
$9 < m \leq 11$	21
$11 < m \leq 13$	19
$13 < m \leq 15$	11
$15 < m \leq 17$	4
$17 < m \leq 19$	1

1.1 Write down the modal class of the data. (1)

1.2 Complete the cumulative frequency column in the table in the ANSWER BOOK. (2)

1.3 Draw a cumulative frequency graph (ogive) for the given data on the grid provided in the ANSWER BOOK. (3)

1.4 Use the graph to determine the median mass for this data. (2)

1.5 The international guideline for the mass of a school bag is that it should not exceed 10% of a learner's body mass.

1.5.1 Calculate the estimated mean mass of the school bags. (2)

1.5.2 The mean mass of this group of learners was found to be 80 kg. On average, are these school bags satisfying the international guideline with regard to mass? Motivate your answer. (2)

[12]

DBE/2024

QUESTION 2

Fifty athletes need to access suitable training facilities. The table below shows the distances, in km, that they need to travel to obtain access to suitable training facilities.

DISTANCE (x km)	NUMBER OF ATHLETES
$0 \leq x < 5$	3
$5 \leq x < 10$	7
$10 \leq x < 15$	20
$15 \leq x < 20$	12
$20 \leq x < 25$	5
$25 \leq x < 30$	3

2.1 Complete the cumulative frequency column provided in the table in the ANSWER BOOK. (2)

2.2 On the grid provided in the ANSWER BOOK, draw a cumulative frequency graph (ogive) to represent the above data. (3)

2.3 Calculate the interquartile range (IQR) of the above data. (2)

2.4 The families of 4 of the athletes above who stay between 15 and 20 km from a suitable training facility, decide to move 10 kilometres closer to the facility. In which interval will the number of athletes increase? (1)

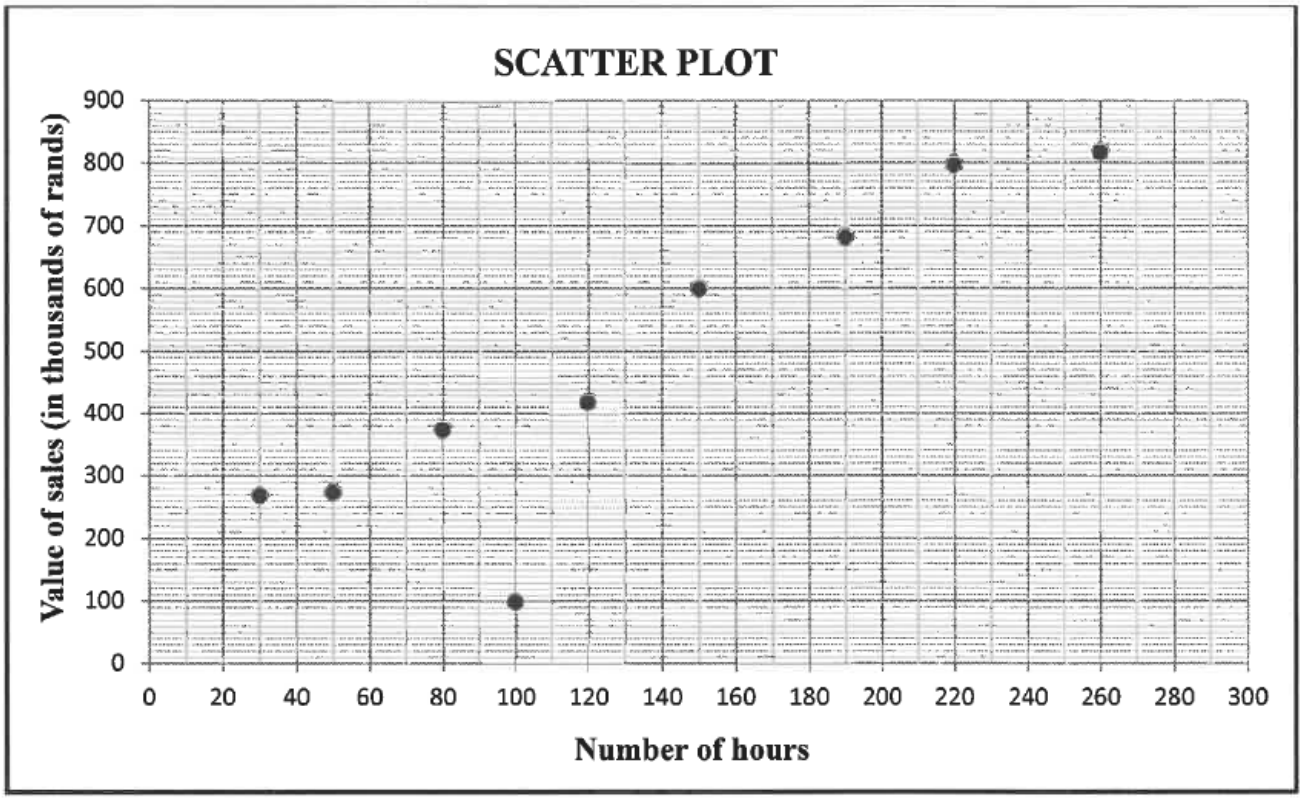
2.5 Calculate the estimated mean distance that the fifty athletes need to travel after the 4 families have moved 10 kilometres closer to the facility. Clearly show ALL working. (3)

[11]

QUESTION 2

The table below shows the number of hours that a sales representative of a company spent with each of his nine clients in one year and the value of the sales (in thousands of rands) for that client.

NUMBER OF HOURS	30	50	80	100	120	150	190	220	260
VALUE OF SALES (IN THOUSANDS OF RANDS)	270	275	376	100	420	602	684	800	820



2.1 Identify an outlier in the data above. (1)

2.2 Calculate the equation of the least squares regression line of the data. (3)

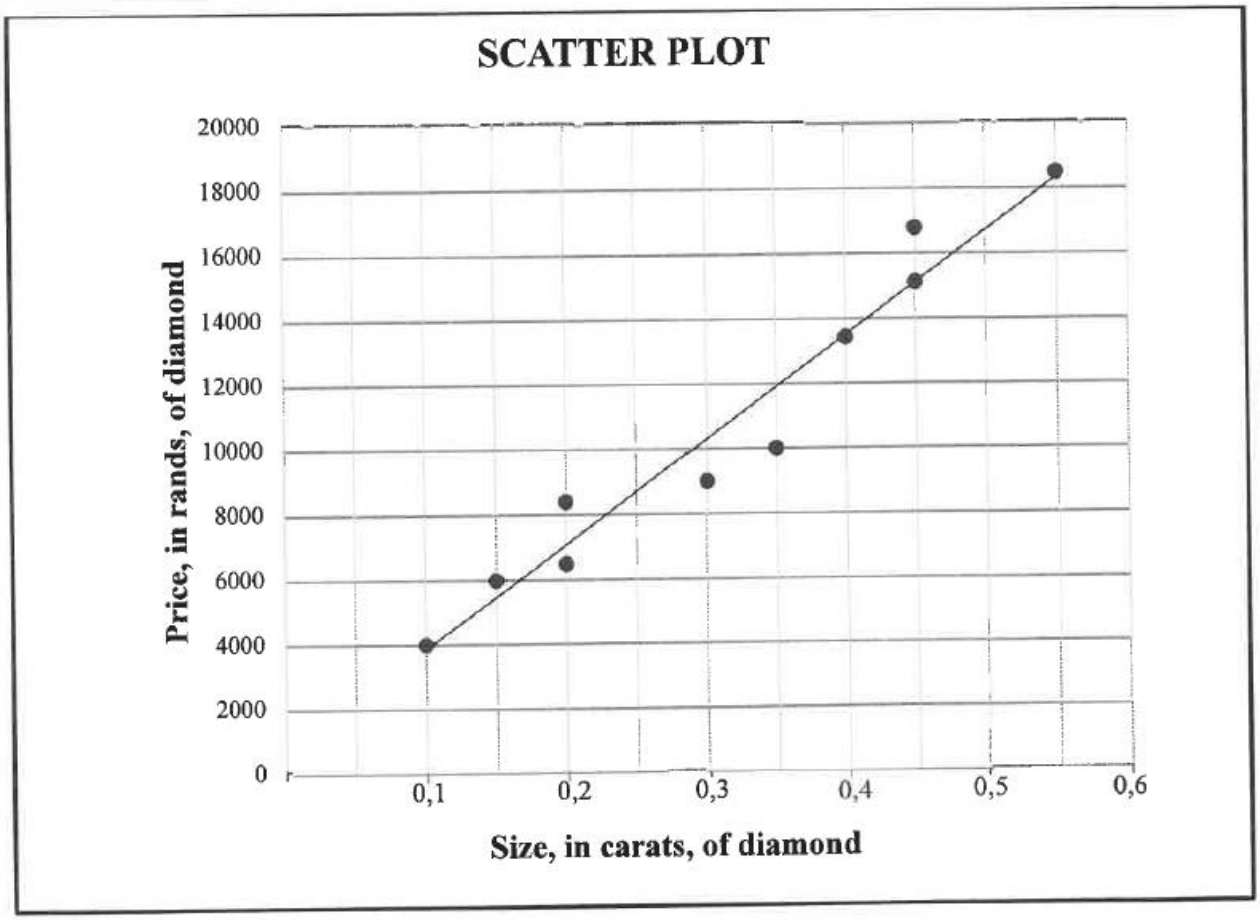
2.3 The sales representative forgot to record the sales of one of his clients. Predict the value of this client's sales (in thousands of rands) if he spent 240 hours with him during the year. (2)

2.4 What is the expected increase in sales for EACH additional hour spent with a client? (2)
[8]

QUESTION 2

The table below shows the size (in carats) and the price (in rands) of 10 diamonds that were sold by a diamond trader. This information is also presented in the scatter plot below. The least squares regression line for the data is drawn.

Size, in carats, of diamond (x)	0,1	0,15	0,2	0,2	0,3	0,35	0,4	0,45	0,45	0,55
Price, in rands, of diamond (y)	4 000	6 000	6 500	8 400	9 000	10 000	13 440	15 120	16 800	18 480



2.1 Determine the equation of the least squares regression line for the data. (3)

2.2 If the trader sold a diamond that was 0,25 carats in size, predict the selling price of this diamond in rands. (2)

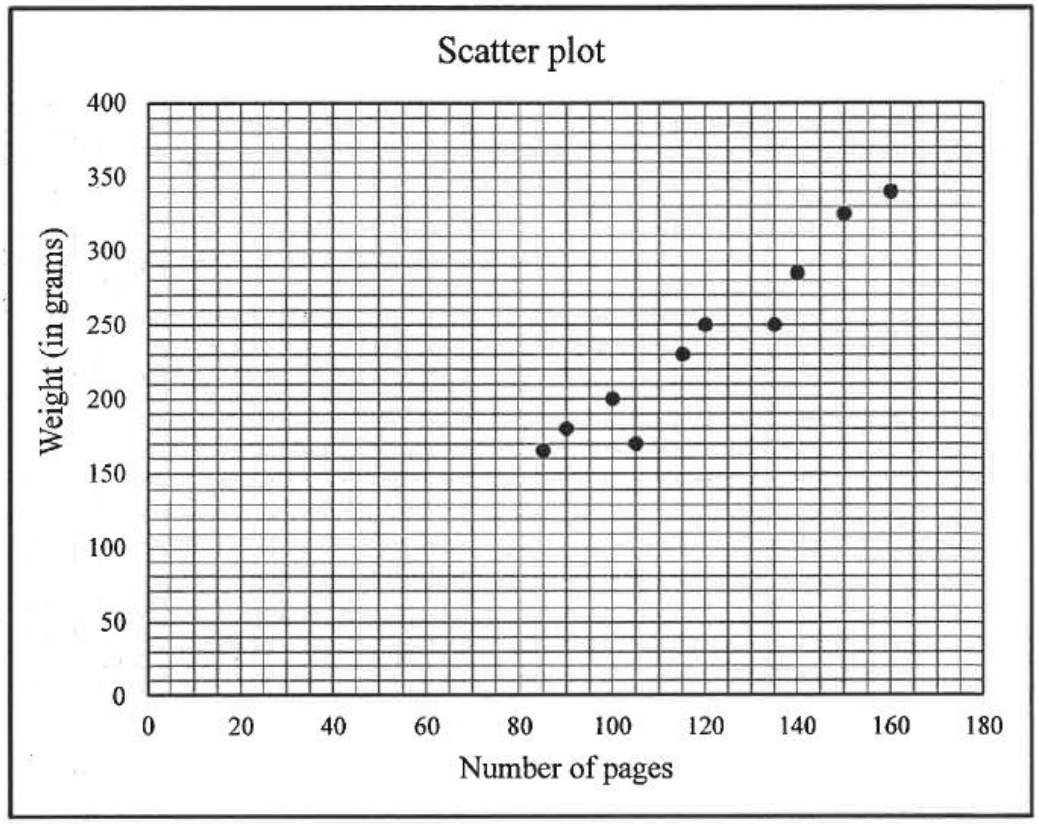
2.3 Calculate the average price increase per 0,05 carat of the diamonds. (2)

2.4 It was later found that the selling price of the 0,35 carat diamond was recorded incorrectly. The correct price is R11 500. When this correction is made to the data set, the correlation between the size and price of these diamonds gets stronger. Explain the reason for this by referring to the given scatter plot. (1)
[8]

QUESTION 1

The number of pages in ten A4 books and their corresponding weights (in grams) are given in the table below. The data is also represented in the scatter plot.

Number of pages (x)	85	150	100	120	90	140	135	105	115	160
Weight (in grams) (y)	165	325	200	250	180	285	250	170	230	340



1.1 Determine the equation of the least squares regression line. (3)

1.2 Draw the least squares regression line on the scatter plot in the ANSWER BOOK. (2)

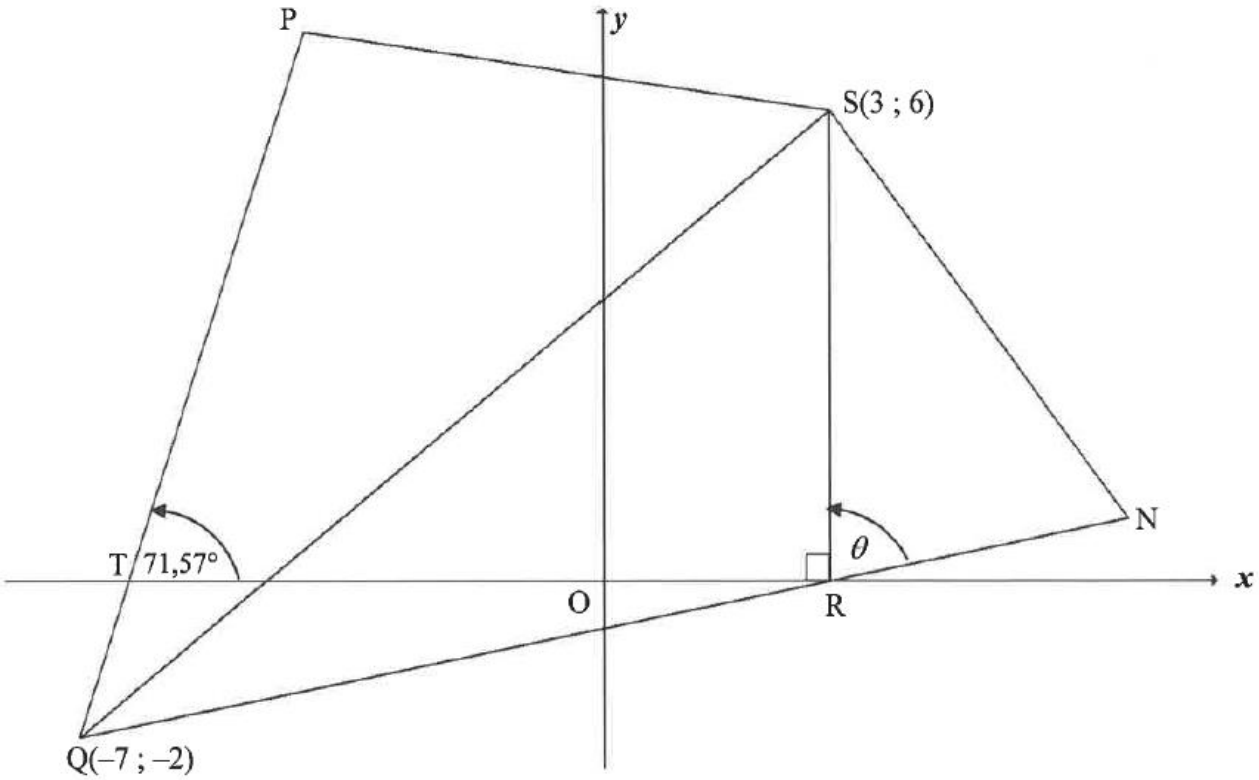
1.3 Predict the weight of an A4 book that has 110 pages. (2)

1.4 Calculate the percentage weight increase between a book with 110 pages and a book with 130 pages. (3)
[10]

DBE/March 2018

QUESTION 3

In the diagram, P, Q(–7 ; –2), R and S(3 ; 6) are vertices of a quadrilateral. R is a point on the x-axis. QR is produced to N such that QR = 2RN. SN is drawn. $P\hat{T}O = 71{,}57°$ and $S\hat{R}N = \theta$.



Determine:

3.1 The equation of SR (1)

3.2 The gradient of QP to the nearest integer (2)

3.3 The equation of QP in the form $y = mx + c$ (2)

3.4 The length of QR. Leave your answer **in surd form**. (2)

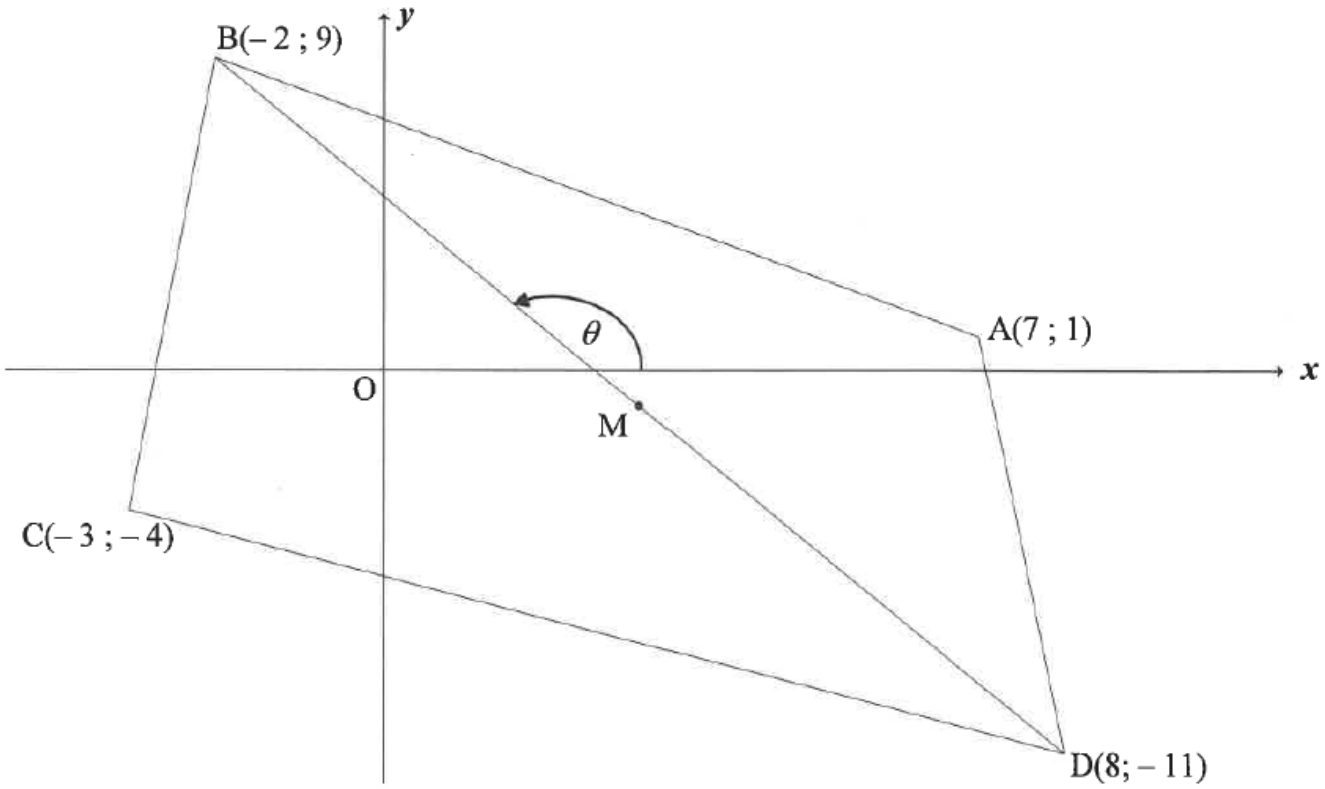
3.5 $\tan(90° - \theta)$ (3)

3.6 The area of ΔRSN, **without using a calculator** (6)

[16]

QUESTION 3

In the diagram, ABCD is a quadrilateral having vertices A(7 ; 1), B(– 2 ; 9), C(– 3 ; – 4) and D(8 ; – 11). M is the midpoint of BD.



3.1 Calculate the gradient of AC. (2)

3.2 Determine:

3.2.1 The equation of AC in the form $y = mx + c$ (2)

3.2.2 Whether M lies on AC (4)

3.3 Prove that BD $\perp$ AC. (3)

3.4 Calculate:

3.4.1 θ, the inclination of BD (2)

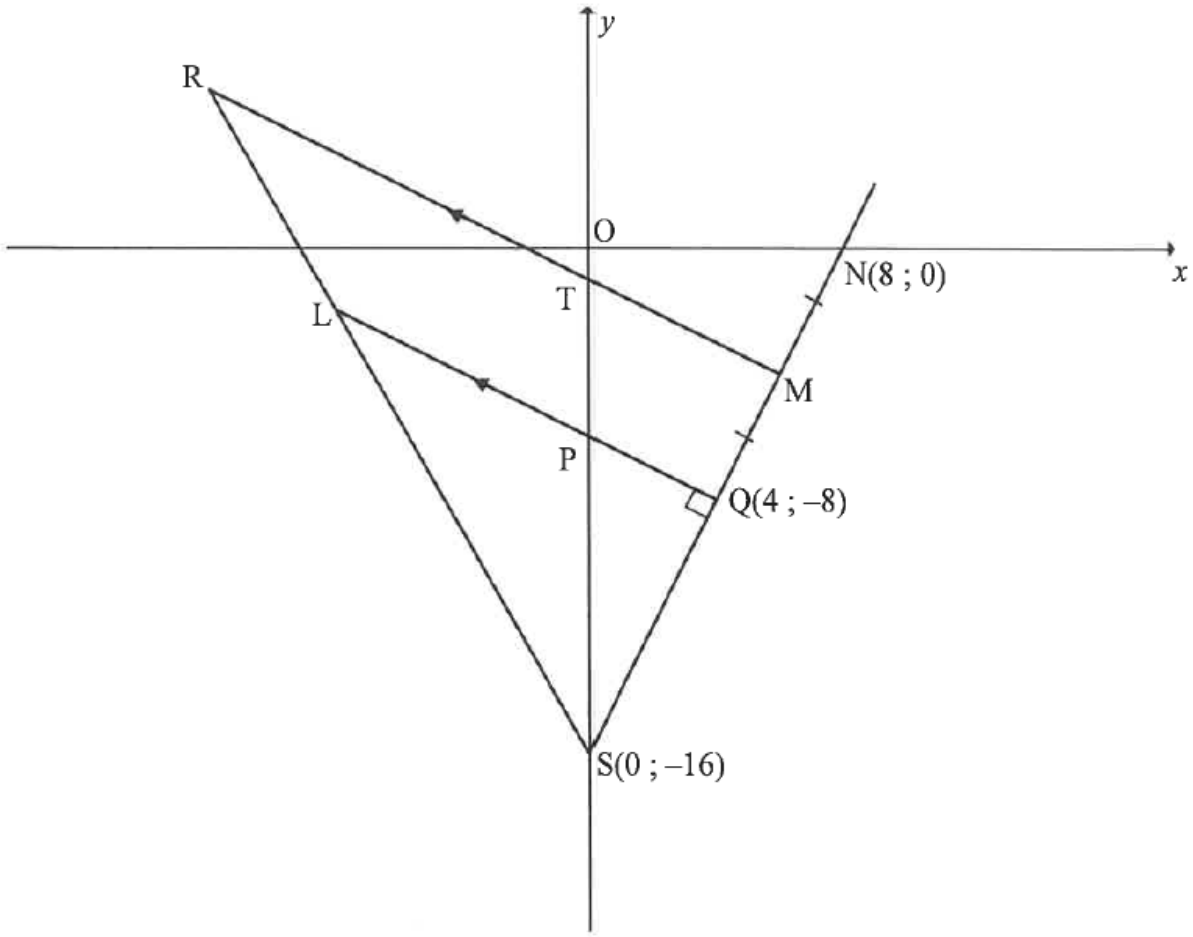
3.4.2 The size of $C\hat{B}D$ (3)

3.4.3 The length of AC (2)

3.4.4 The area of ABCD (5)
[23]

QUESTION 3

In the diagram, S(0 ; –16), L and Q(4 ; –8) are the vertices of ΔSLQ having LQ perpendicular to SQ. SL and SQ are produced to points R and M respectively such that RM ∥ LQ. SM produced cuts the x-axis at N(8 ; 0). QM = MN. T and P are the y-intercepts of RM and LQ respectively.



3.1 Calculate the coordinates of M. (2)

3.2 Calculate the gradient of NS. (2)

3.3 Show that the equation of line LQ is $y=-\frac{1}{2}x-6$. (3)

3.4 Determine the equation of a circle having centre at O, the origin, and also passing through S. (2)

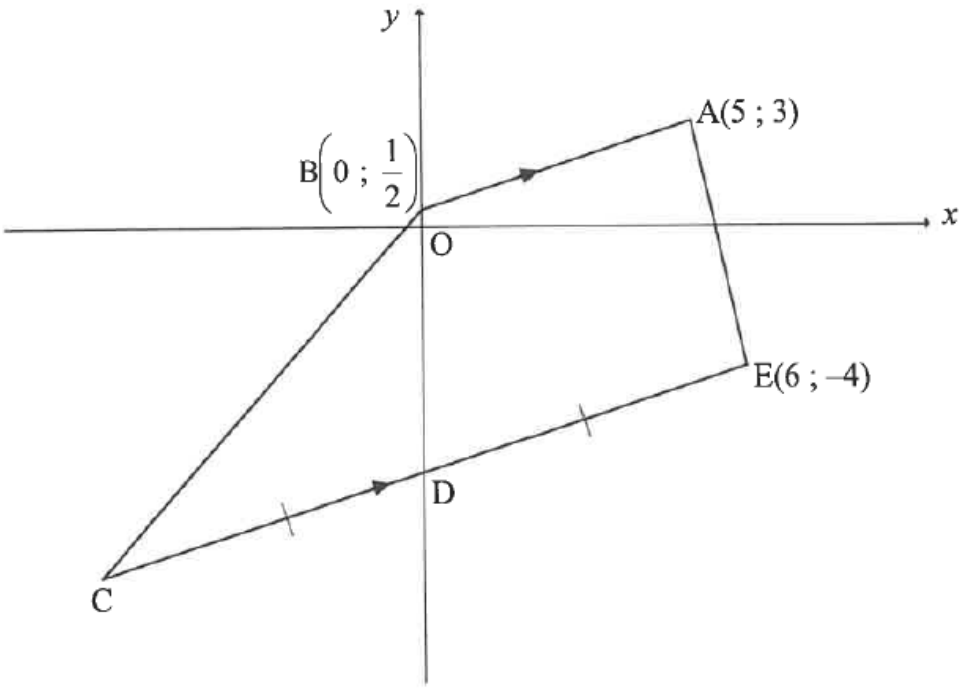
3.5 Calculate the coordinates of T. (3)

3.6 Determine $\frac{LS}{RS}$. (3)

3.7 Calculate the area of PTMQ. (4)
[19]

QUESTION 3

In the diagram, A(5 ; 3), $B\left(0 ; \frac{1}{2}\right)$, C and E(6 ; –4) are the vertices of a trapezium having BA ∥ CE. D is the y-intercept of CE and CD = DE.



3.1 Calculate the gradient of AB. (2)

3.2 Determine the equation of CE in the form $y = mx + c$. (3)

3.3 Calculate the:

3.3.1 Coordinates of C (3)

3.3.2 Area of quadrilateral ABCD (4)

3.4 If point K is the reflection of E in the y-axis:

3.4.1 Write down the coordinates of K (2)

3.4.2 Calculate the:

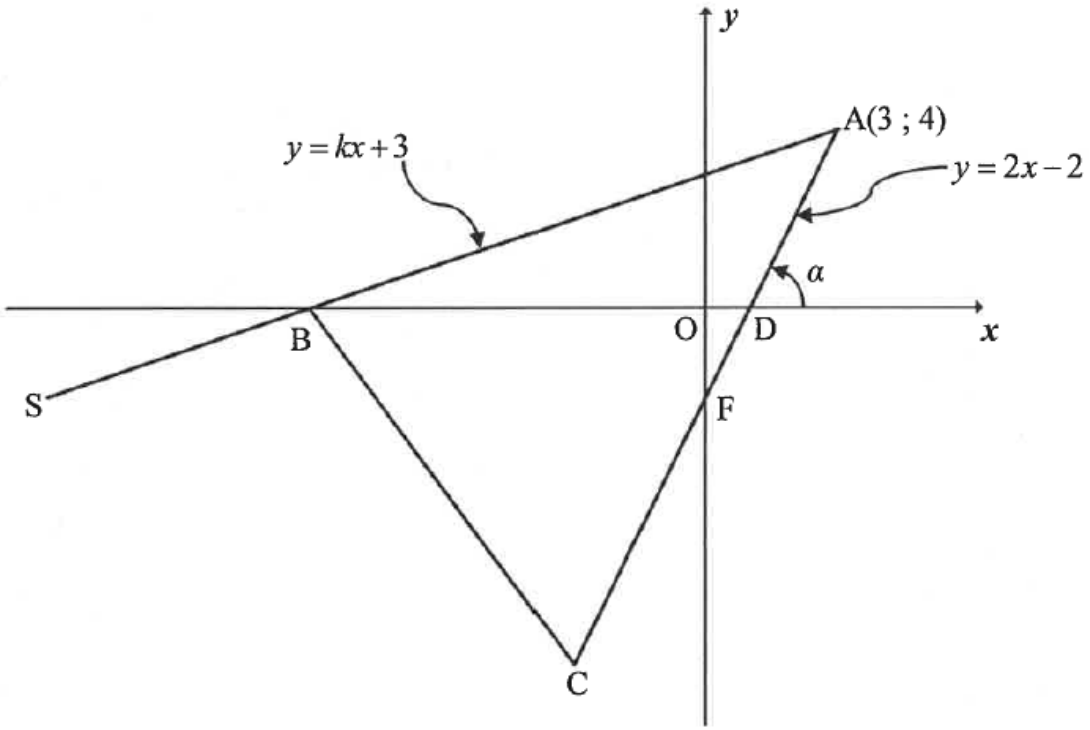
(a) Perimeter of ΔKEC (4)

(b) Size of $K\hat{C}E$ (3)

[21]

QUESTION 3

In the diagram, A(3 ; 4), B and C are vertices of ΔABC. AB is produced to S. D and F are the x- and y-intercepts of AC respectively. F is the midpoint of AC and the angle of inclination of AC is α. The equation of AB is $y = kx + 3$ and the equation of AC is $y = 2x - 2$.



3.1 Show that $k = \frac{1}{3}$. (1)

3.2 Calculate the coordinates of B, the x-intercept of line AS. (2)

3.3 Calculate the coordinates of C. (4)

3.4 Determine the equation of the line parallel to BC and passing through S(–15 ; –2). Write your answer in the form $y = mx + c$. (5)

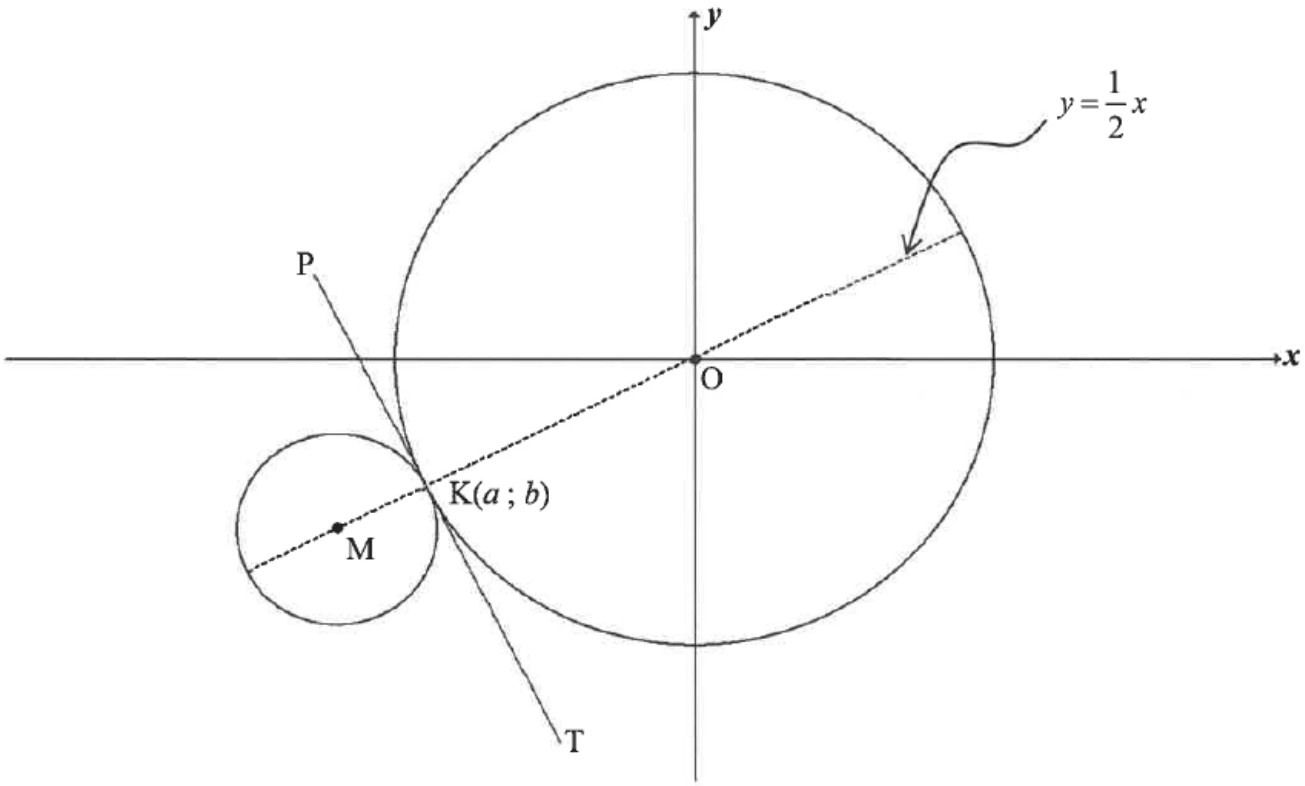
3.5 Calculate the size of $B\hat{A}C$. (5)

3.6 If it is further given that the length of AC is $6\sqrt{5}$ units, calculate the value of $\frac{\text{Area of } \Delta ABD}{\text{Area of } \Delta ASC}$. (5)

[22]

QUESTION 4

In the diagram, PKT is a common tangent to both circles at K(a ; b). The centres of both circles lie on the line $y = \frac{1}{2}x$. The equation of the circle centred at O is $x^2 + y^2 = 180$. The radius of the circle is three times that of the circle centred at M.



4.1 Write down the length of OK **in surd form.** (1)

4.2 Show that K is the point (–12 ; –6). (4)

4.3 Determine:

4.3.1 The equation of the common tangent, PKT, in the form $y = mx + c$ (3)

4.3.2 The coordinates of M (6)

4.3.3 The equation of the smaller circle in the form $(x-a)^2 + (y-b)^2 = r^2$ (2)

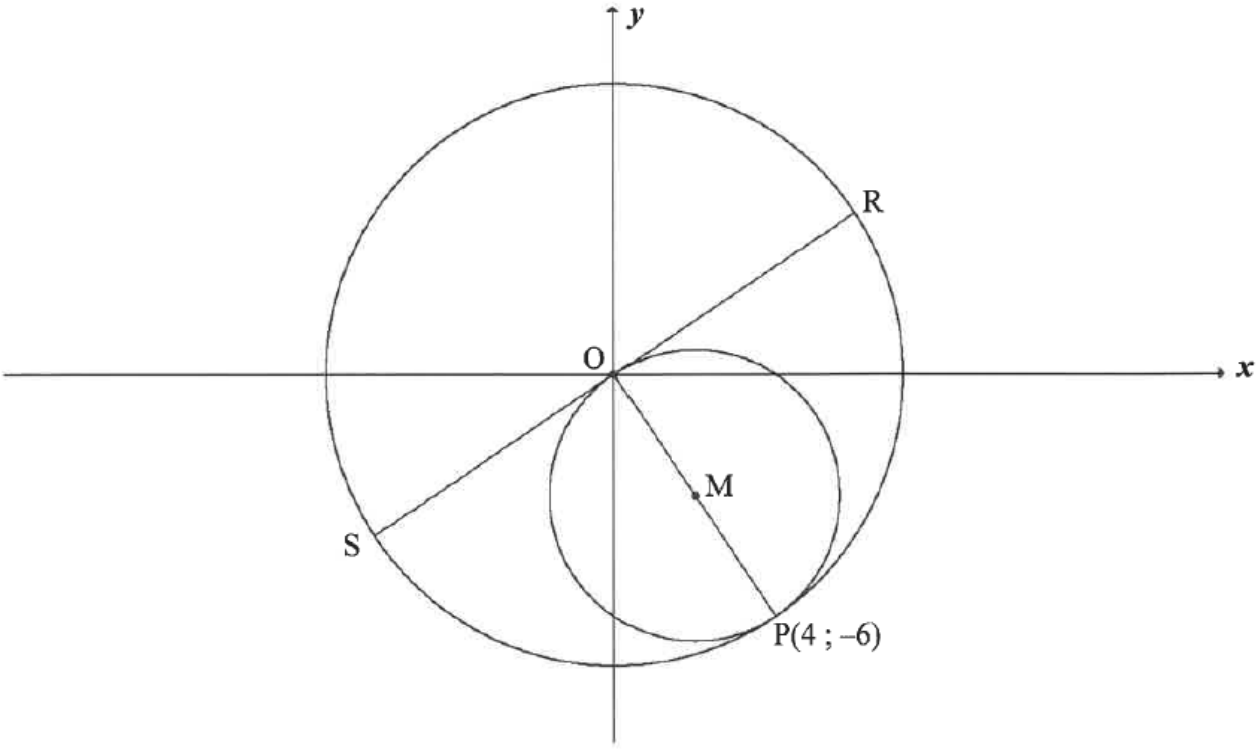
4.4 For which value(s) of r will another circle, with equation $x^2 + y^2 = r^2$, intersect the circle centred at M at two distinct points? (3)

4.5 Another circle, $x^2 + y^2 + 32x + 16y + 240 = 0$, is drawn. Prove by calculation that this circle does NOT cut the circle with centre M(–16 ; –8). (5)

[24]

QUESTION 4

In the diagram, a circle having centre at the origin passes through P(4 ; – 6). PO is the diameter of a smaller circle having centre at M. The diameter RS of the larger circle is a tangent to the smaller circle at O.



4.1 Calculate the coordinates of M. (2)

4.2 Determine the equation of:

4.2.1 The large circle (2)

4.2.2 The small circle in the form $x^2 + y^2 + Cx + Dy + E = 0$ (3)

4.2.3 The equation of RS in the form $y = mx + c$ (3)

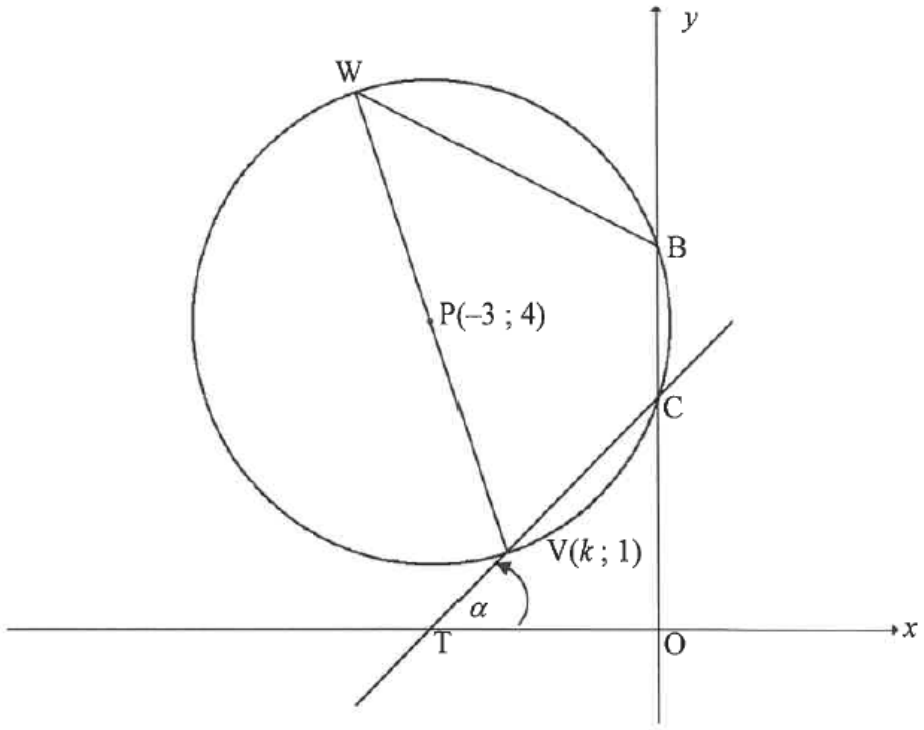
4.3 Determine the length of chord NR, where N is the reflection of R in the y-axis. (4)

4.4 The circle with centre at M is reflected about the x-axis to form another circle centred at K. Calculate the length of the common chord of these two circles. (3)

[17]

QUESTION 4

In the diagram, P(–3 ; 4) is the centre of the circle. V(k ; 1) and W are the endpoints of a diameter. The circle intersects the y-axis at B and C. BCVW is a cyclic quadrilateral. CV is produced to intersect the x-axis at T. $O\hat{T}C = \alpha$.



4.1 The radius of the circle is $\sqrt{10}$. Calculate the value of k if point V is to the right of point P. Clearly show ALL calculations. (5)

4.2 The equation of the circle is given as $x^2 + 6x + y^2 - 8y + 15 = 0$. Calculate the length of BC. (4)

4.3 If $k = -2$, calculate the size of:

4.3.1 α (3)

4.3.2 $V\hat{W}B$ (2)

4.4 A new circle is obtained when the given circle is reflected about the line $y = 1$.

Determine the:

4.4.1 Coordinates of Q, the centre of the new circle (2)

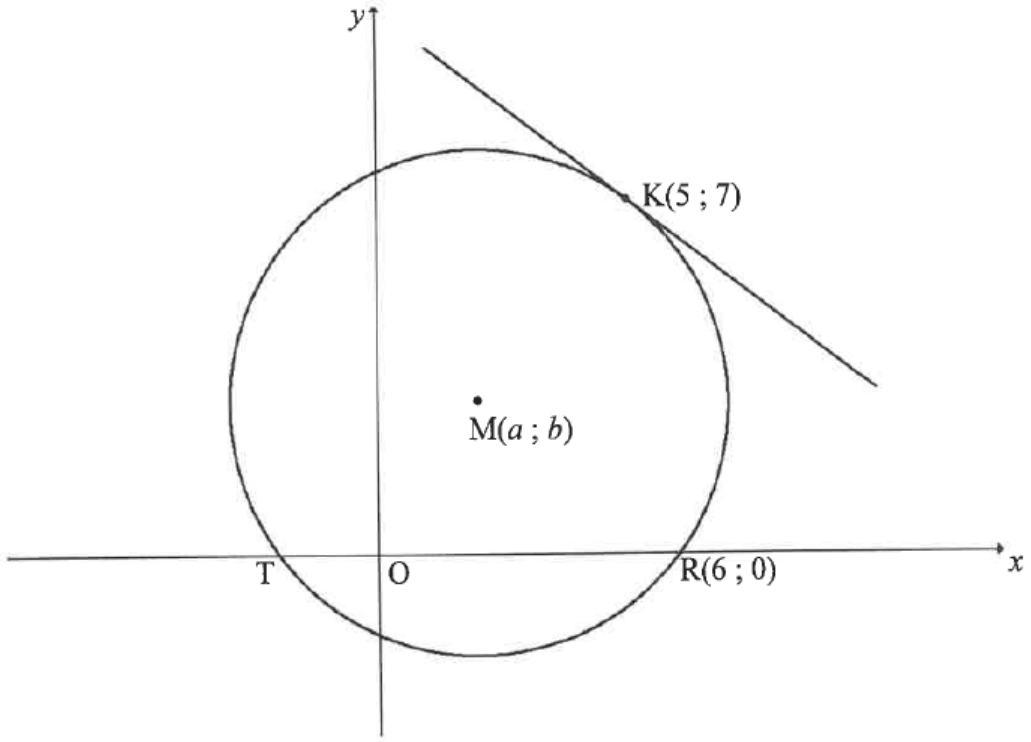
4.4.2 Equation of the new circle in the form $(x-a)^2 + (y-b)^2 = r^2$ (2)

4.4.3 Equations of the lines drawn parallel to the y-axis and passing through the points of intersection of the two circles (2)

[20]

QUESTION 4

In the diagram, the circle centred at M(a ; b) is drawn. T and R(6 ; 0) are the x-intercepts of the circle. A tangent is drawn to the circle at K(5 ; 7).



4.1 M is a point on the line $y = x + 1$.

4.1.1 Write b in terms of a. (1)

4.1.2 Calculate the coordinates of M. (5)

4.2 If the coordinates of M are (2 ; 3), calculate the length of:

4.2.1 The radius of the circle (2)

4.2.2 TR (2)

4.3 Determine the equation of the tangent to the circle at K. Write your answer in the form $y = mx + c$. (5)

4.4 A horizontal line is drawn as a tangent to the circle M at the point N(c ; d), where $d < 0$.

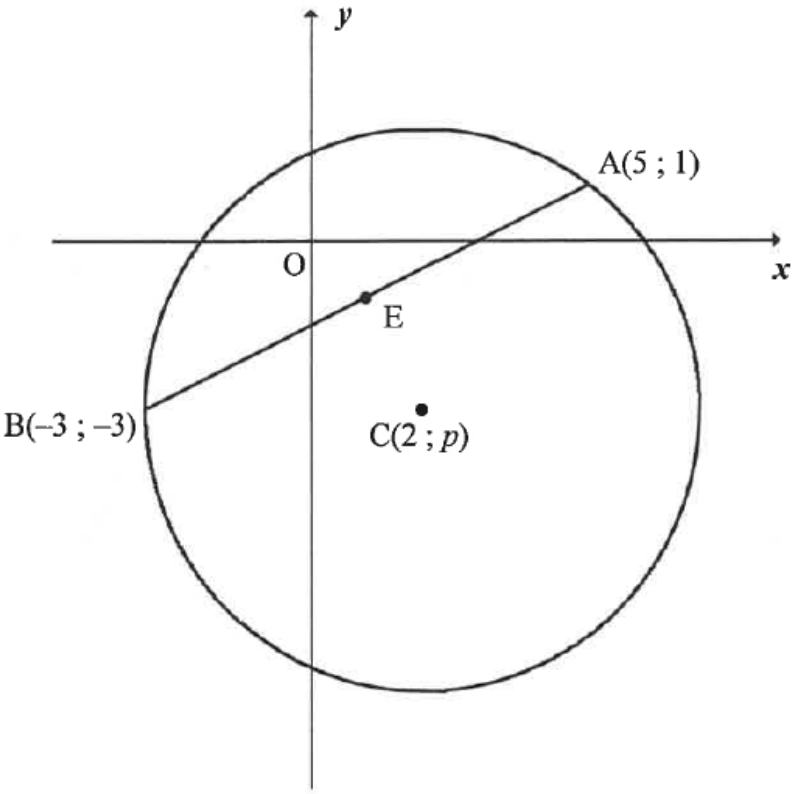
4.4.1 Write down the coordinates of N. (2)

4.4.2 Determine the equation of the circle centred at N and passing through T. Write your answer in the form $(x-a)^2 + (y-b)^2 = r^2$. (3)

[20]

QUESTION 4

In the diagram, the circle centred at C(2 ; p) is drawn. A(5 ; 1) and B$(-3;-3)$ are points on the circle. E is the midpoint of AB.



4.1 Calculate the coordinates of E, the midpoint of AB. (2)

4.2 Calculate the length of AB. Leave your answer in surd form. (1)

4.3 Determine the equation of the perpendicular bisector of AB in the form $y = mx + c$. (4)

4.4 Show that $p = -3$. (1)

4.5 Show, by calculation, that the equation of the circle is $x^2 + y^2 - 4x + 6y - 12 = 0$ (4)

4.6 Calculate the values of t for which the straight line $y = tx + 8$ will not intersect the circle. (6)

[18]

DBE/March 2018

QUESTION 5

5.1 If $\cos 2\theta = -\frac{5}{6}$, where $2\theta \in [180°;270°]$, calculate, **without using a calculator**, the values in simplest form of:

5.1.1 $\sin 2\theta$ (4)

5.1.2 $\sin^2 \theta$ (3)

5.2 Simplify $\sin(180° - x).\cos(-x) + \cos(90° + x).\cos(x - 180°)$ to a single trigonometric ratio. (6)

5.3 Determine the value of $\sin 3x.\cos y + \cos 3x.\sin y$ if $3x + y = 270°$. (2)

5.4 Given: $2\cos x = 3\tan x$

5.4.1 Show that the equation can be rewritten as $2\sin^2 x + 3\sin x - 2 = 0$. (3)

5.4.2 Determine the general solution of x if $2\cos x = 3\tan x$. (5)

5.4.3 Hence, determine two values of y, $144° \leq y \leq 216°$, that are solutions of $2\cos 5y = 3\tan 5y$. (4)

5.5 Consider: $g(x) = -4\cos(x + 30°)$

5.5.1 Write down the maximum value of $g(x)$. (1)

5.5.2 Determine the range of $g(x) + 1$. (2)

5.5.3 The graph of g is shifted $60°$ to the left and then reflected about the x-axis to form a new graph h. Determine the equation of h in its simplest form. (3)

[33]

QUESTION 5

5.1 In ΔMNP, $\hat{N} = 90°$ and $\sin M = \frac{15}{17}$.
Determine, **without using a calculator**:

5.1.1 tan M (3)

5.1.2 The length of NP if MP = 51 (2)

5.2 Simplify to a single term: $\cos(x-360°).\sin(90°+x)+\cos^2(-x)-1$ (4)

5.3 Consider: $\sin(2x+40°)\cos(x+30°)-\cos(2x+40°)\sin(x+30°)$

5.3.1 Write as a single trigonometric term in its simplest form. (2)

5.3.2 Determine the general solution of the following equation:

$$\sin(2x+40°)\cos(x+30°)-\cos(2x+40°)\sin(x+30°)=\cos(2x-20°)$$ (7)

[18]

QUESTION 6

6.1 If $90^0 < A < 360^0$ and $\tan A = \frac{2}{3}$, determine without the use of a calculator.

6.1.1 sin A (3)

6.1.2 cos 2A – sin 2A (4)

6.2 Given that $\sin x = t$, express the following in terms of t, without the use of calculator.

6.2.1 $\cos(x-90^0)$ (2)

6.2.2 $\sin 2x$ (3)

[12]

QUESTION 7

7.1 Simplify: $\frac{\cos^2(90+x)}{(-\tan x)(\sin 2x)}$ (4)

7.2 Calculate the general solution of :
$1-\cos 2\theta = 8\sin\theta.\sin 2\theta$ (6)

[10]

DBE/2021

QUESTION 5

5.1 Simplify the expression to a **single trigonometric term:**

$$\tan(-x).\cos x.\sin(x-180°)-1$$ (5)

5.2 Given: $\cos 35° = m$

Without using a calculator, determine the value of EACH of the following in terms of m:

5.2.1 $\cos 215°$ (2)

5.2.2 $\sin 20°$ (3)

5.3 Determine the general solution of:

$$\cos 4x.\cos x + \sin x.\sin 4x = -0,7$$ (4)

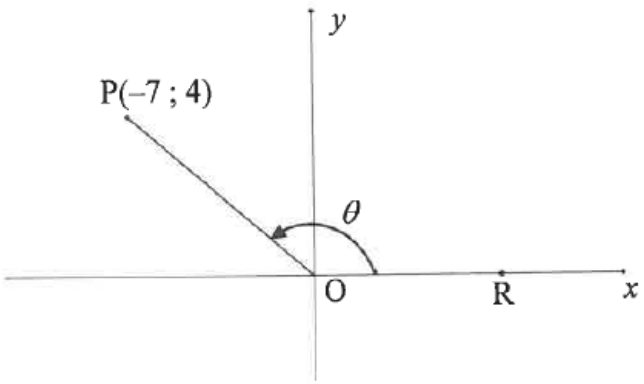
5.4 Prove the identity: $\dfrac{\sin 4x.\cos 2x - 2\cos 4x.\sin x.\cos x}{\tan 2x} = \cos^2 x - \sin^2 x$ (4)

[18]

DBE/2022

QUESTION 5

5.1 In the diagram below, P(–7 ; 4) is a point in the Cartesian plane. R is a point on the positive x-axis such that obtuse $\hat{POR} = \theta$.



Calculate, **without using a calculator**, the:

5.1.1 Length OP (2)

5.1.2 Value of:

(a) $\tan\theta$ (1)

(b) $\cos(\theta - 180°)$ (2)

5.2 Determine the general solution of: $\sin x\cos x+\sin x=3\cos^2 x+3\cos x$ (7)

5.3 Given the identity: $\dfrac{\sin 3x}{1-\cos 3x}=\dfrac{1+\cos 3x}{\sin 3x}$

5.3.1 Prove the identity given above. (3)

5.3.2 Determine the values of x, in the interval $x\in[0°\,;\,60°]$, for which the identity will be undefined. (3)

[18]

DBE/2024

QUESTION 5

5.1 If $\sin 40°=p$, write EACH of the following in terms of p.

5.1.1 $\sin 220°$ (2)

5.1.2 $\cos^2 50°$ (2)

5.1.3 $\cos(-80°)$ (3)

5.2 Given: $\tan x(1-\cos^2 x)+\cos^2 x=\dfrac{(\sin x+\cos x)(1-\sin x\cos x)}{\cos x}$

5.2.1 Prove the above identity. (5)

5.2.2 For which values of x, in the interval $x\in[-180°\,;180°]$, will the identity be undefined? (3)

5.3 Given the expression: $\dfrac{\sin 150°+\cos^2 x-1}{2}$

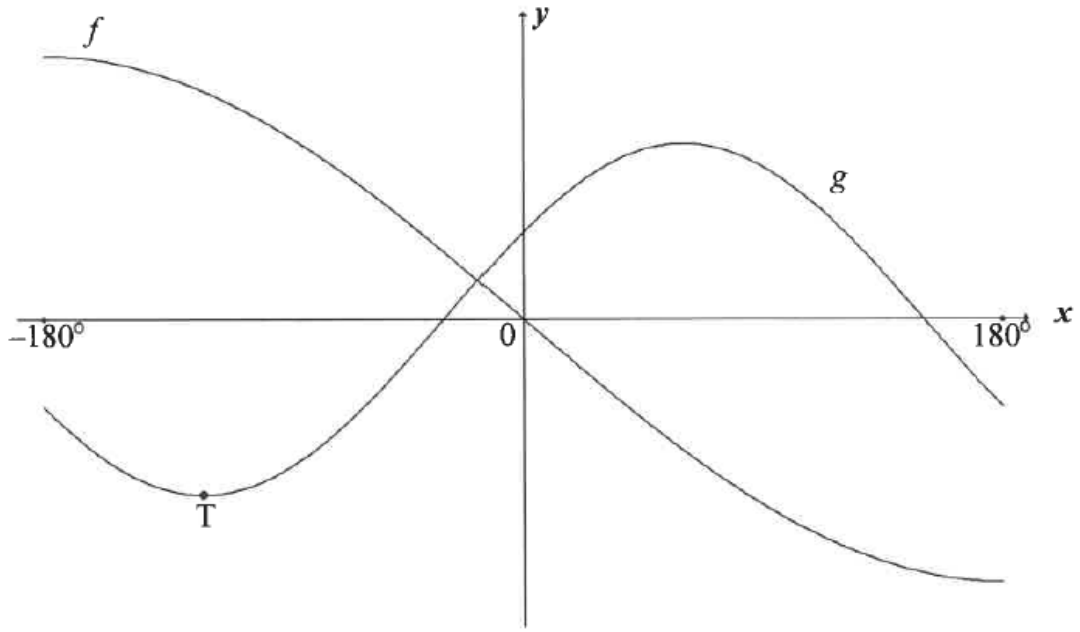
5.3.1 **Without using a calculator**, simplify the expression given above to a single trigonometric term in terms of $\cos 2x$. (6)

5.3.2 Hence, determine the general solution of $\dfrac{\sin 150°+\cos^2 x-1}{2}=\dfrac{1}{25}$ (5)

[26]

QUESTION 6

In the diagram, the graphs of $f(x) = -3\sin\frac{x}{2}$ and $g(x) = 2\cos(x - 60°)$ are drawn in the interval $x \in [-180°;180°]$. T$(p\,;q)$ is a turning point of g with $p < 0$.



6.1 Write down the period of f. (1)

6.2 Write down the range of g. (2)

6.3 Calculate $f(p) - g(p)$. (3)

6.4 Use the graphs to determine the value(s) of x in the interval $x \in [-180°;180°]$ for which:

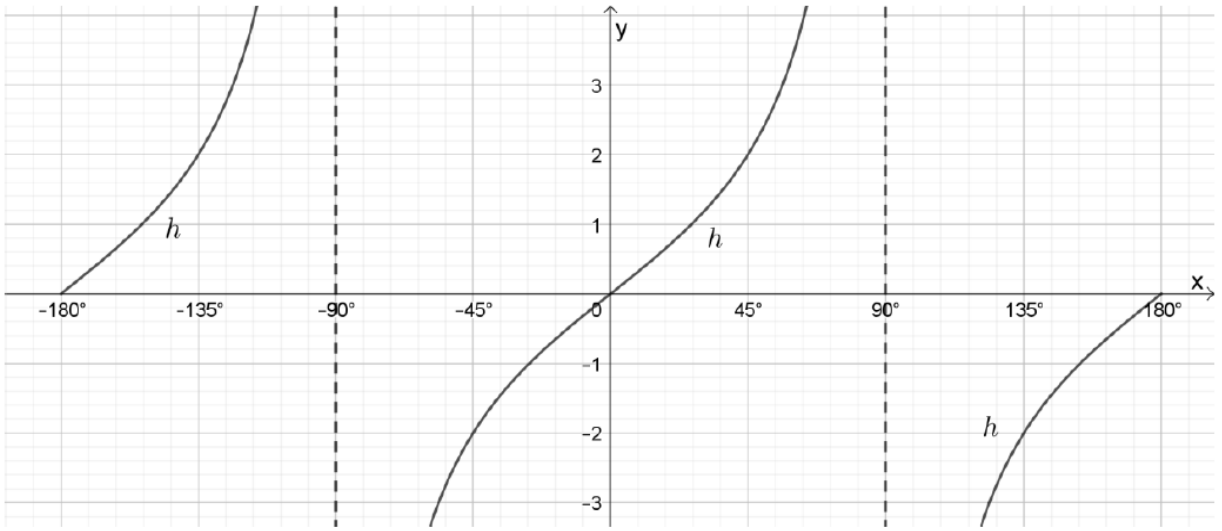
6.4.1 $g(x) > 0$ (3)

6.4.2 $g(x).g'(x) > 0$ (4)

[13]

QUESTION 8

The graph of $h(x) = a\tan x$; for $x \in [-180°; 180°]$, $x \neq -90°$, is sketched below.



8.1 Determine the value of a. (2)

8.2 If $f(x) = \cos(x + 45°)$, sketch the graph of f for $x \in [-180°; 180°]$, on the diagram provided in your ANSWER BOOK. (4)

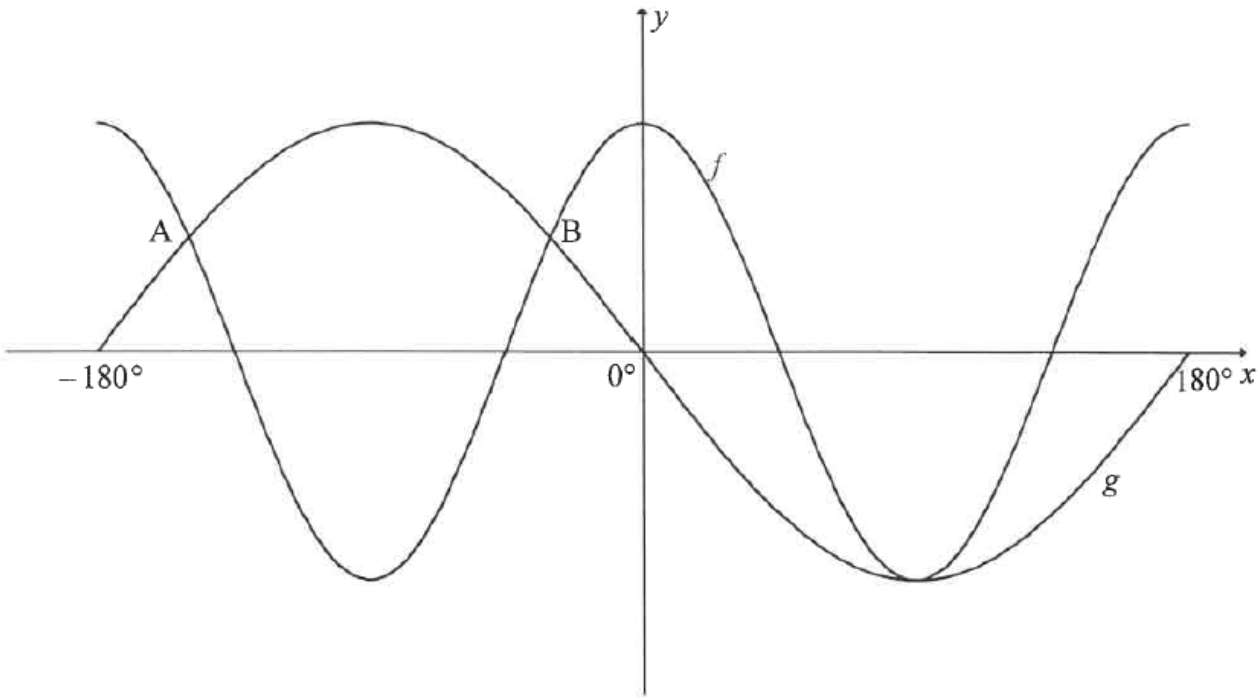
8.3 How many solutions does the equation $h(x) = f(x)$ have in the domain $[-180°; 180°]$? (1)

[7]

DBE/2021

QUESTION 6

In the diagram below, the graphs of $f(x) = \cos 2x$ and $g(x) = -\sin x$ are drawn for the interval $x \in [-180° ; 180°]$ A and B are two points of intersection of f and g.



6.1 **Without using a calculator**, determine the values of x for which $\cos 2x = -\sin x$ in the interval $x \in [-180° ; 180°]$. (6)

6.2 Use the graphs above to answer the following questions:

6.2.1 How many degrees apart are points A and B from each other? (2)

6.2.2 For which values of x in the given interval will $f'(x).g'(x) > 0$? (2)

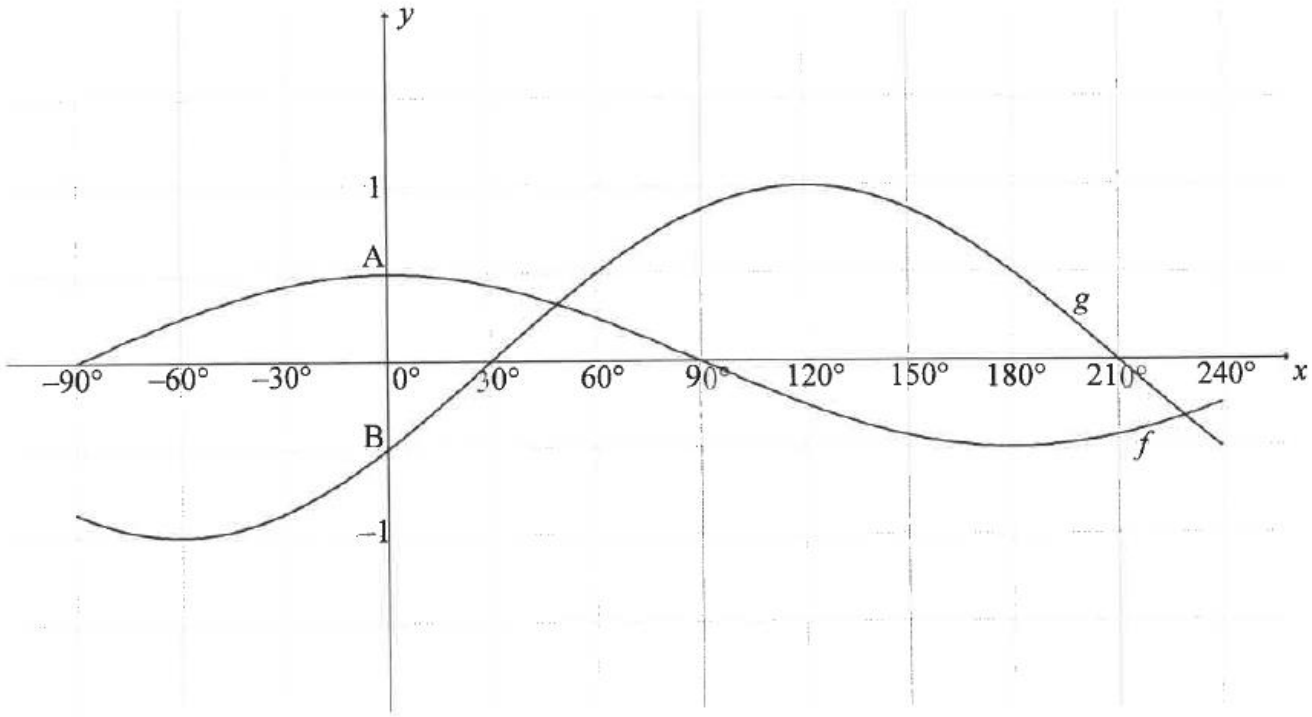
6.2.3 Determine the values of k for which $\cos 2x + 3 = k$ will have no solution. (3)

[13]

DBE/2022

QUESTION 7

In the diagram below, the graphs of $f(x) = \frac{1}{2}\cos x$ and $g(x) = \sin(x - 30°)$ are drawn for the interval $x \in [-90° ; 240°]$. A and B are the y-intercepts of f and g respectively.



7.1 Determine the length of AB. (2)

7.2 Write down the range of $3f(x) + 2$. (2)

7.3 Read off from the graphs a value of x for which $g(x) - f(x) = \frac{\sqrt{3}}{2}$. (2)

7.4 For which values of x, in the interval $x \in [-90° ; 240°]$, will:

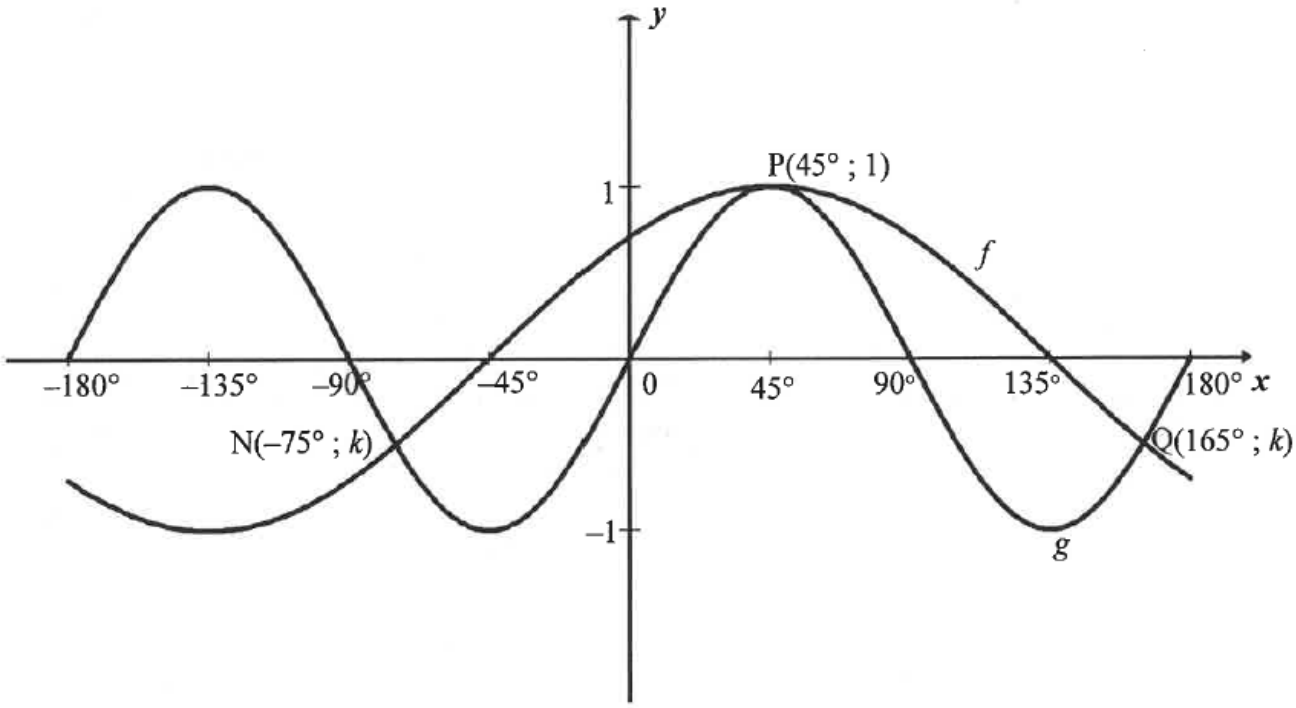
7.4.1 $f(x).g(x) > 0$ (2)

7.4.2 $g'(x - 5°) > 0$ (2)

[10]

QUESTION 6

In the diagram, the graphs of $f(x)=\cos(x+a)$ and $g(x)=\sin 2x$ are drawn for the interval $x\in[-180°;180°]$. The graphs intersect at N(–75° ; k), P(45° ; 1) and Q(165° ; k). P is also a turning point of both graphs.



6.1 Write down the period of f. (1)

6.2 Write down the amplitude of g. (1)

6.3 Write down the value of a. (1)

6.4 Calculate the value of k, the y-coordinate of N and Q, **without the use of a calculator**. (2)

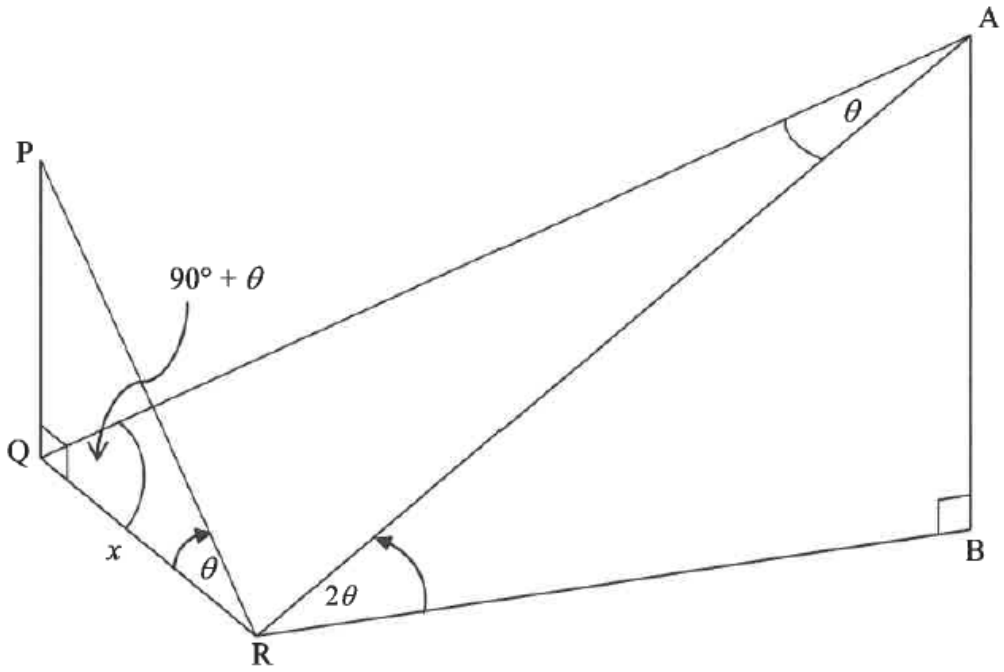
6.5 Calculate the value of x if $g(x+60°)=f(x+60°)$ and $x\in[-45°;0°]$. (1)

6.6 **Without using a calculator,** determine the number of solutions the equation $\sqrt{2}\sin 2x=\sin x+\cos x$ has in the interval $x\in[-90°;90°]$. Clearly show ALL working. (4)

[10]

QUESTION 6

PQ and AB are two vertical towers.
From a point R in the same horizontal plane as Q and B, the angles of elevation to P and A are θ and 2θ respectively.
$A\hat{Q}R = 90° + \theta$, $Q\hat{A}R = \theta$ and $QR = x$.



6.1 Determine in terms of x and θ:

6.1.1 QP (2)

6.1.2 AR (2)

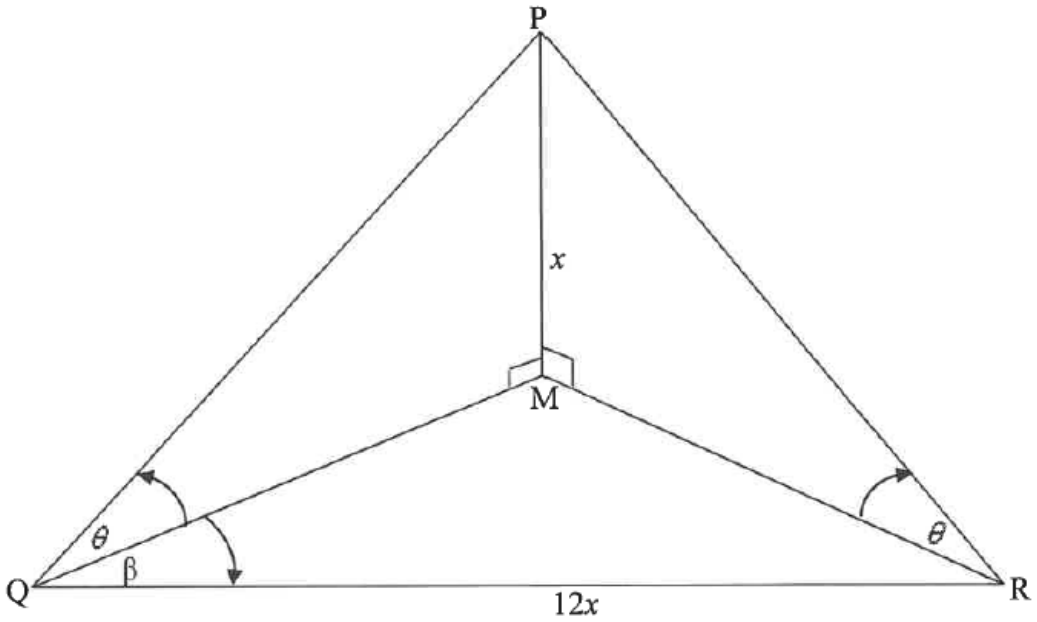
6.2 Show that $AB = 2x\cos^2\theta$ (4)

6.3 Determine $\frac{AB}{QP}$ if $\theta = 12°$. (2)

[10]

QUESTION 7

The captain of a boat at sea, at point Q, notices a lighthouse PM directly north of his position. He determines that the angle of elevation of P, the top of the lighthouse, from Q is θ and the height of the lighthouse is x metres. From point Q the captain sails $12x$ metres in a direction β degrees east of north to point R. From point R, he notices that the angle of elevation of P is also θ. Q, M and R lie in the same horizontal plane.



7.1 Write QM in terms of x and θ. (2)

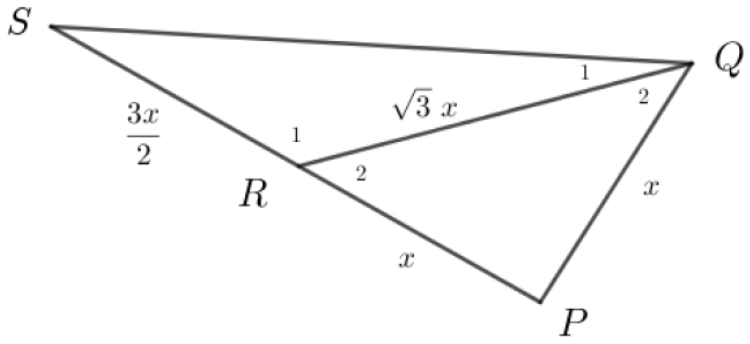
7.2 Prove that $\tan\theta = \frac{\cos\beta}{6}$. (4)

7.3 If $\beta = 40°$ and QM = 60 metres, calculate the height of the lighthouse **to the nearest metre.** (3)

[9]

QUESTION 9

Triangle PQS represents a certain area of a park. R is a point on line PS such that QR divides the area of the park into two triangular parts, as shown below.
PQ = PR = x units, RS = $\frac{3x}{2}$ units and RQ = $\sqrt{3}x$ units.

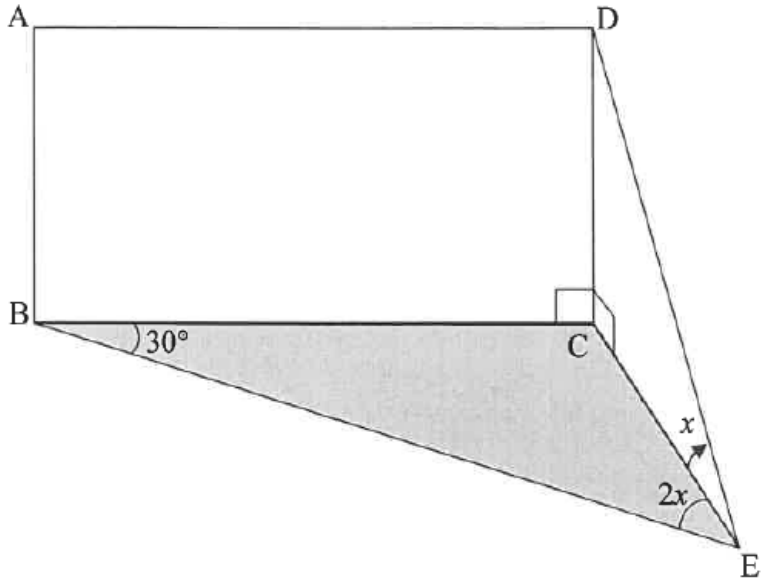


9.1 Calculate the size of $\hat{P}$. (4)

9.2 Determine the area of triangle PQR in terms of x. (2)
[6]

QUESTION 7

Points B, C and E lie in the same horizontal plane. ABCD is a rectangular piece of board. CDE is a triangular piece of board having a right angle at C. Each piece of board is placed perpendicular to the horizontal plane and joined along DC, as shown in the diagram. The angle of elevation from E to D is x. $B\hat{E}C = 2x$ and $E\hat{B}C = 30°$.



7.1 Show that $DC = \dfrac{BC}{4\cos^2 x}$ (6)

7.2 If $x = 30°$, show that the area of $ABCD = 3AB^2$. (3)

[9]

QUESTION 8

FIGURE I shows a ramp leading to the entrance of a building. B, C and D lie on the same horizontal plane. The perpendicular height (AC) of the ramp is 0,5 m and the angle of elevation from B to A is 15°. The entrance of the building (AE) is 0,915 m wide.

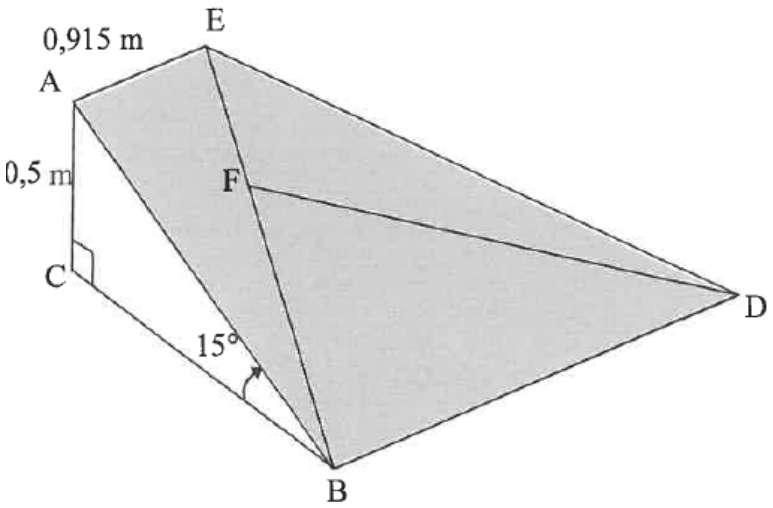


FIGURE I

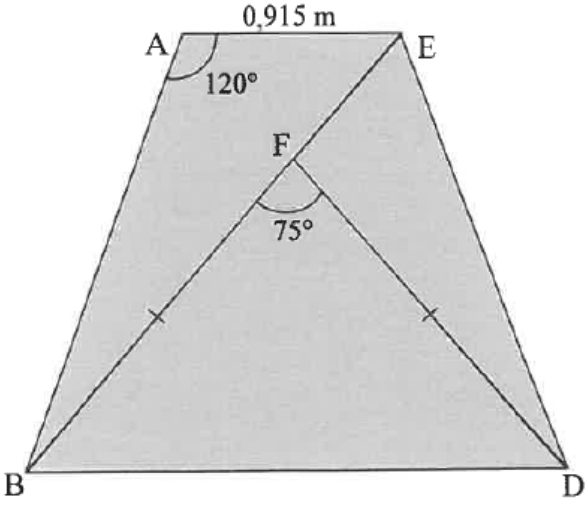


FIGURE II (top view)

8.1 Calculate the length of AB. (2)

8.2 Figure II shows the top view of the ramp. The area of the top of the ramp is divided into three triangles, as shown in the diagram.

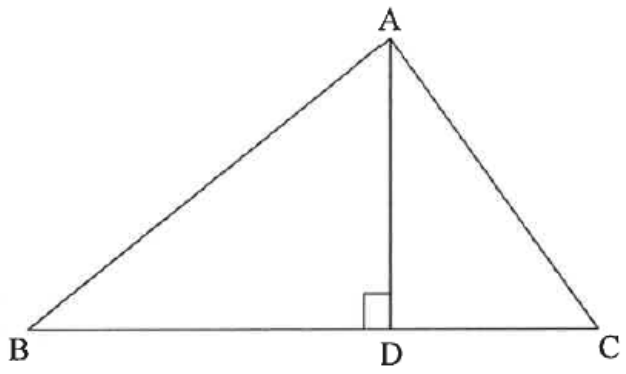
If $B\hat{A}E = 120°$, calculate the length of BE. (3)

8.3 Calculate the area of ΔBFD if $B\hat{F}D = 75°$, $BF = FD$ and $BF = \frac{5}{7}BE$. (3)

[8]

QUESTION 7

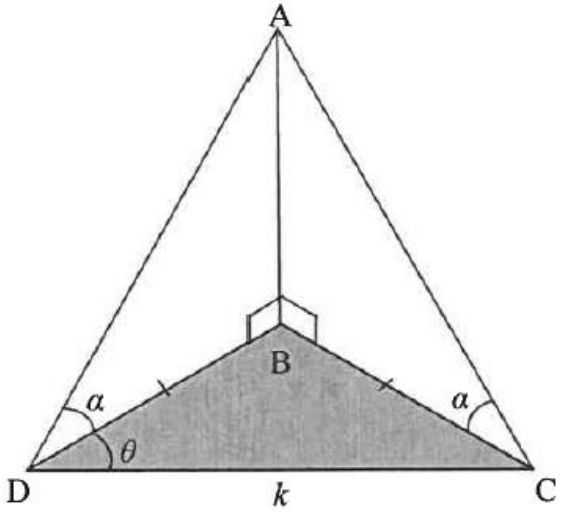
7.1 In the diagram, ΔABC is drawn. AD is drawn such that $AD \perp BC$.



7.1.1 Use the diagram above to determine AD in terms of $\sin\hat{B}$ (2)

7.1.2 Hence, prove that the area of $\Delta ABC = \frac{1}{2}(BC)(AB)\sin\hat{B}$ (1)

7.2 In the diagram, points B, C and D lie in the same horizontal plane. $A\hat{D}B = A\hat{C}B = \alpha$, $C\hat{D}B = \theta$ and $DC = k$ units. BD = BC.



7.2.1 Prove that AD = AC (2)

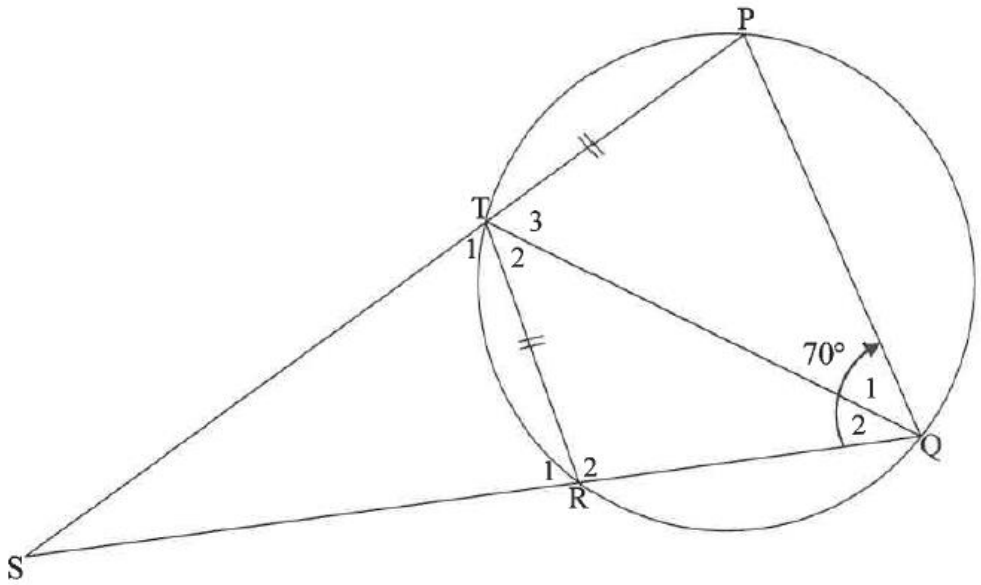
7.2.2 Prove that $BD = \frac{k}{2\cos\theta}$ (3)

7.2.3 Determine the area of ΔBCD in terms of k and a single trigonometric ratio of θ. (3)
[11]

DBE/March 2018

QUESTION 7

In the diagram, PQRT is a cyclic quadrilateral in a circle such that PT = TR. PT and QR are produced to meet in S. TQ is drawn. $S\hat{Q}P = 70°$



7.1 Calculate, with reasons, the size of:

7.1.1 $\hat{T}_1$ (2)

7.1.2 $\hat{Q}_1$ (2)

7.2 If it is further given that PQ || TR:

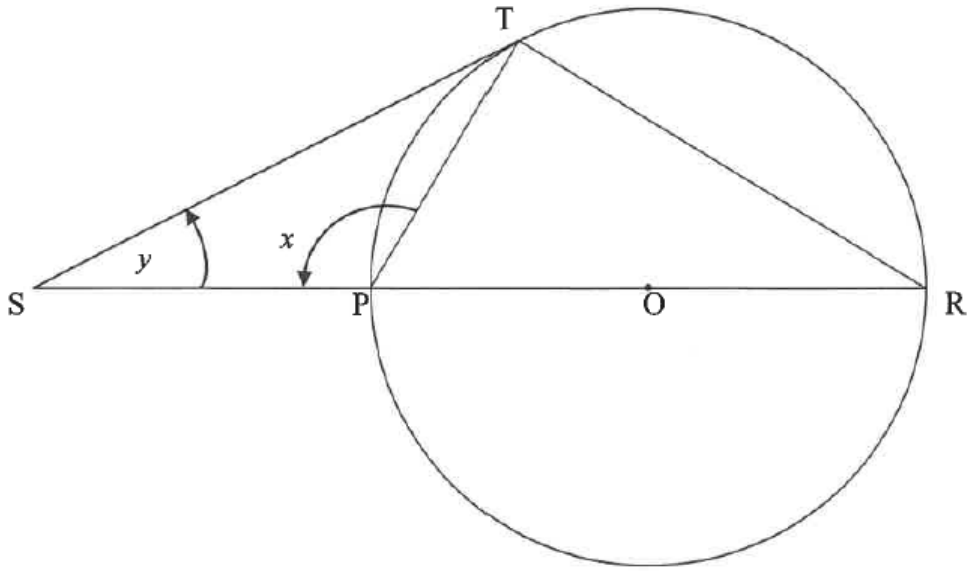
7.2.1 Calculate, with reasons, the size of $\hat{T}_2$ (2)

7.2.2 Prove that $\frac{TR}{TS} = \frac{RQ}{RS}$ (2)

[8]

QUESTION 8

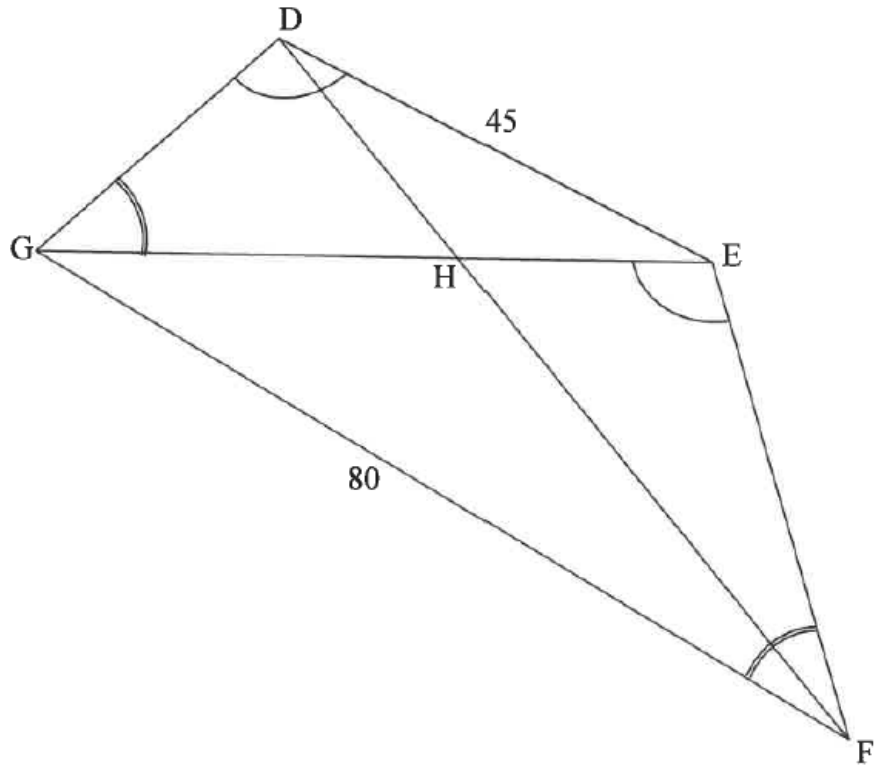
In the diagram, PR is a diameter of the circle with centre O. ST is a tangent to the circle at T and meets RP produced at S. $S\hat{P}T = x$ and $\hat{S} = y$.



Determine, with reasons, y in terms of x. **[6]**

QUESTION 9

In the diagram, DEFG is a quadrilateral with DE = 45 and GF = 80. The diagonals GE and DF meet in H. $G\hat{D}E = F\hat{E}G$ and $D\hat{G}E = E\hat{F}G$.



9.1 Give a reason why $\Delta DEG \;|||\; \Delta EGF$. (1)

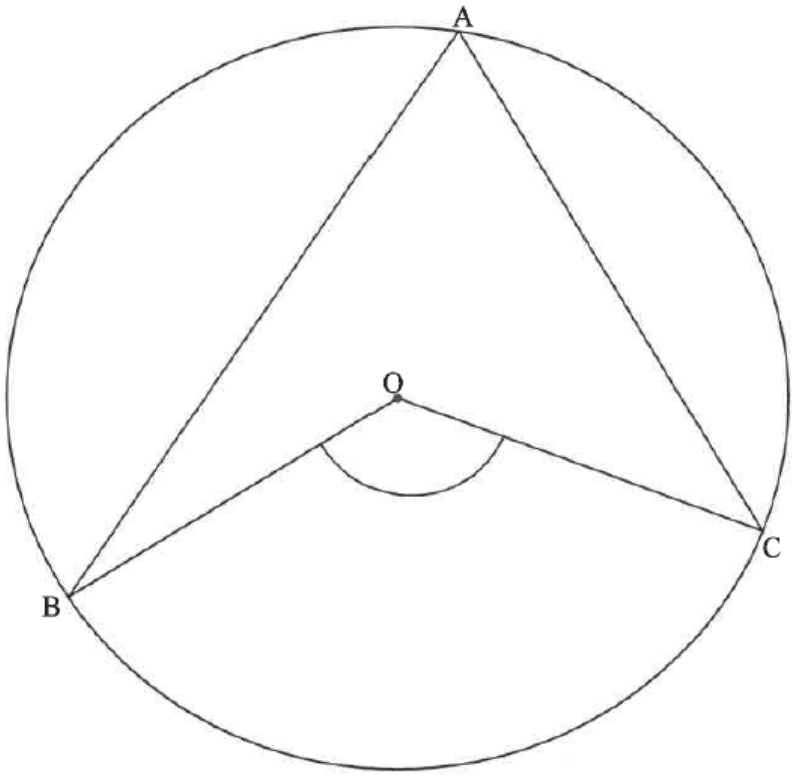
9.2 Calculate the length of GE. (3)

9.3 Prove that $\Delta DEH \;|||\; \Delta FGH$. (3)

9.4 Hence, calculate the length of GH. (3)
[10]

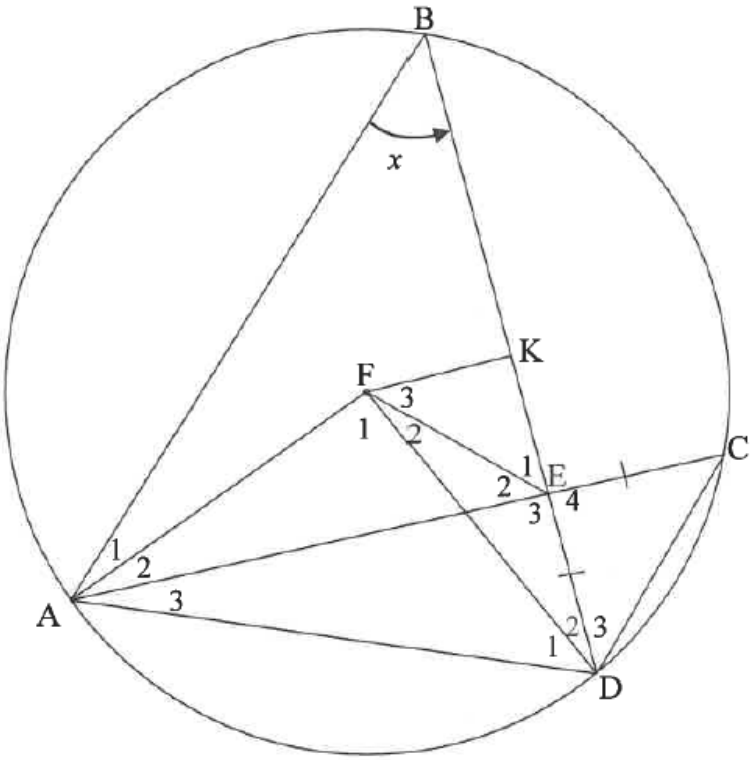
QUESTION 10

10.1 In the diagram, O is the centre of the circle with A, B and C drawn on the circle.



Prove the theorem which states that $B\hat{O}C = 2\hat{A}$. (5)

10.2 In the diagram, the circle with centre F is drawn. Points A, B, C and D lie on the circle. Chords AC and BD intersect at E such that EC = ED. K is the midpoint of chord BD. FK, AB, CD, AF, FE and FD are drawn. Let $\hat{B} = x$.



10.2.1 Determine, with reasons, the size of EACH of the following in terms of x:

(a) $\hat{F}_1$ (2)

(b) $\hat{C}$ (2)

10.2.2 Prove, with reasons, that AFED is a cyclic quadrilateral. (4)

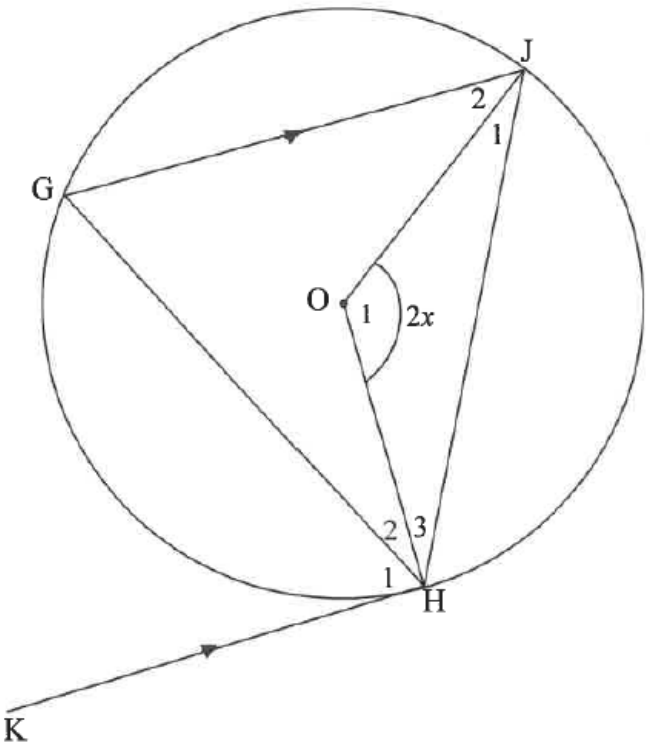
10.2.3 Prove, with reasons, that $\hat{F}_3 = x$. (6)

10.2.4 If area $\Delta AEB = 6{,}25 \times$ area ΔDEC, calculate $\frac{AE}{ED}$. (5)

[24]

QUESTION 8

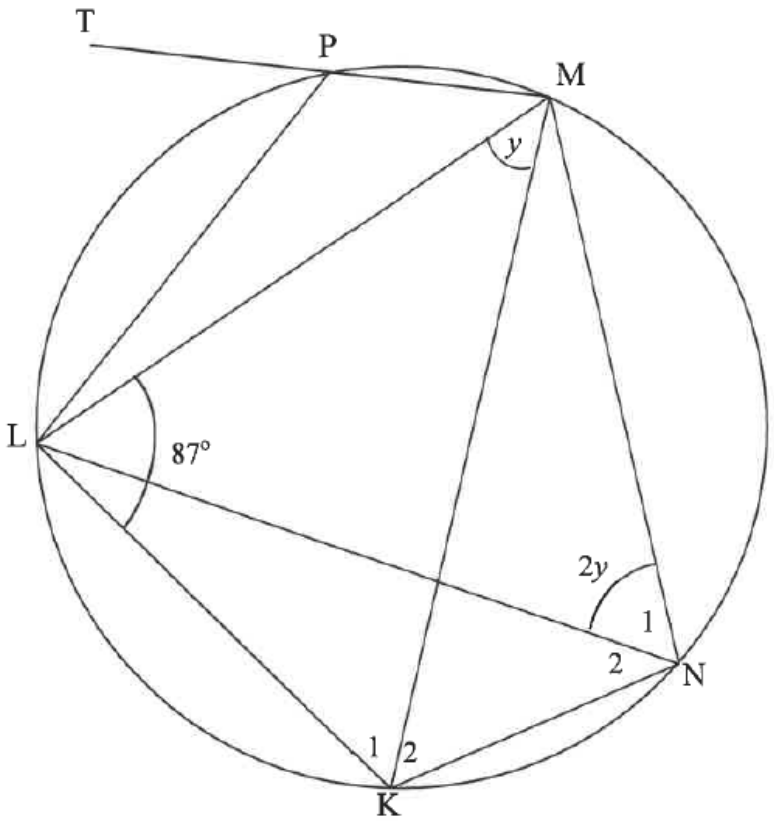
8.1 In the diagram, O is the centre of the circle. Radii OH and OJ are drawn. A tangent is drawn from K to touch the circle at H. ΔHGJ is drawn such that GJ || KH. $\hat{O}_1 = 2x$.



8.1.1 Name, giving reasons, THREE angles, each equal to x. (5)

8.1.2 Prove that $\hat{H}_2 = \hat{H}_3$. (3)

8.2 In the diagram, KLMN is a cyclic quadrilateral with $K\hat{L}M = 87°$. Diagonals LN and MK are drawn. P is a point on the circle and MP is produced to T, a point outside the circle. Chord LP is drawn. $L\hat{M}K = y$ and $\hat{N}_1 = 2y$.



8.2.1 Name, giving a reason, another angle equal to y. (2)

8.2.2 Calculate, giving reasons, the size of:

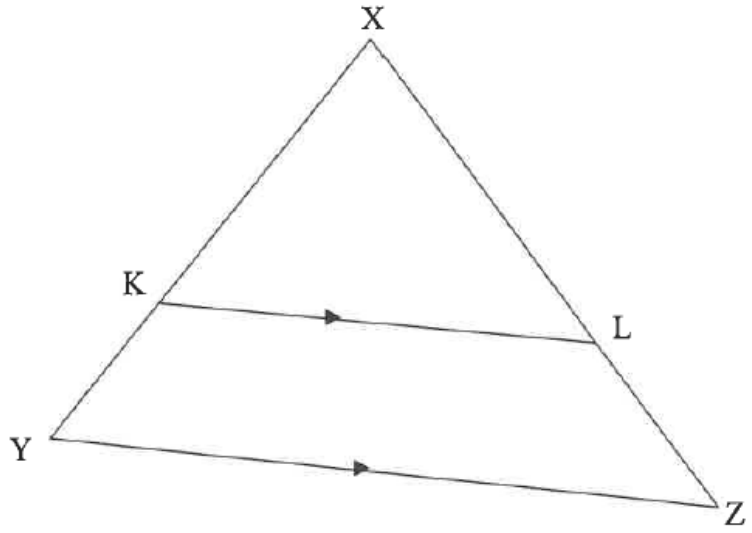
(a) y (3)

(b) $T\hat{P}L$ (2)

[15]

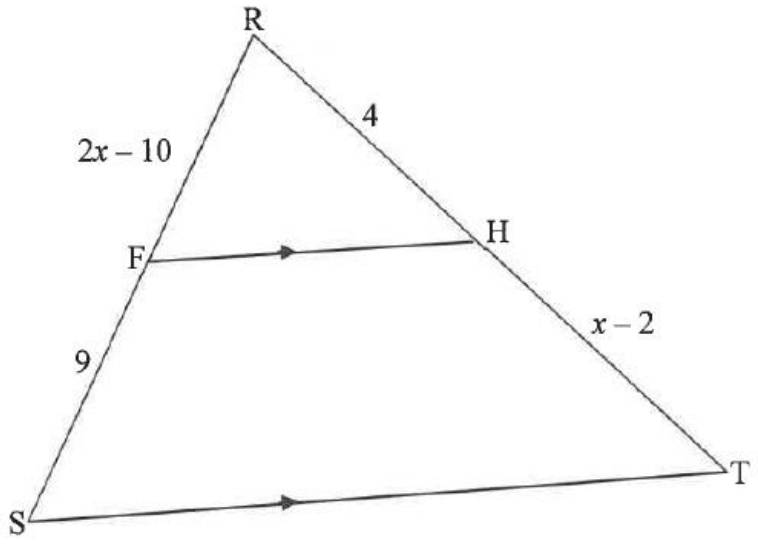
QUESTION 9

9.1 Use the diagram to prove the theorem which states that a line drawn parallel to one side of a triangle divides the other two sides proportionally, that is prove that $\frac{XK}{KY}=\frac{XL}{LZ}$.



(5)

9.2 In ΔRST, F is a point on RS and H is a point on RT such that $FH \parallel ST$. $RF = 2x - 10$, $FS = 9$, $RH = 4$ and $HT = x - 2$.



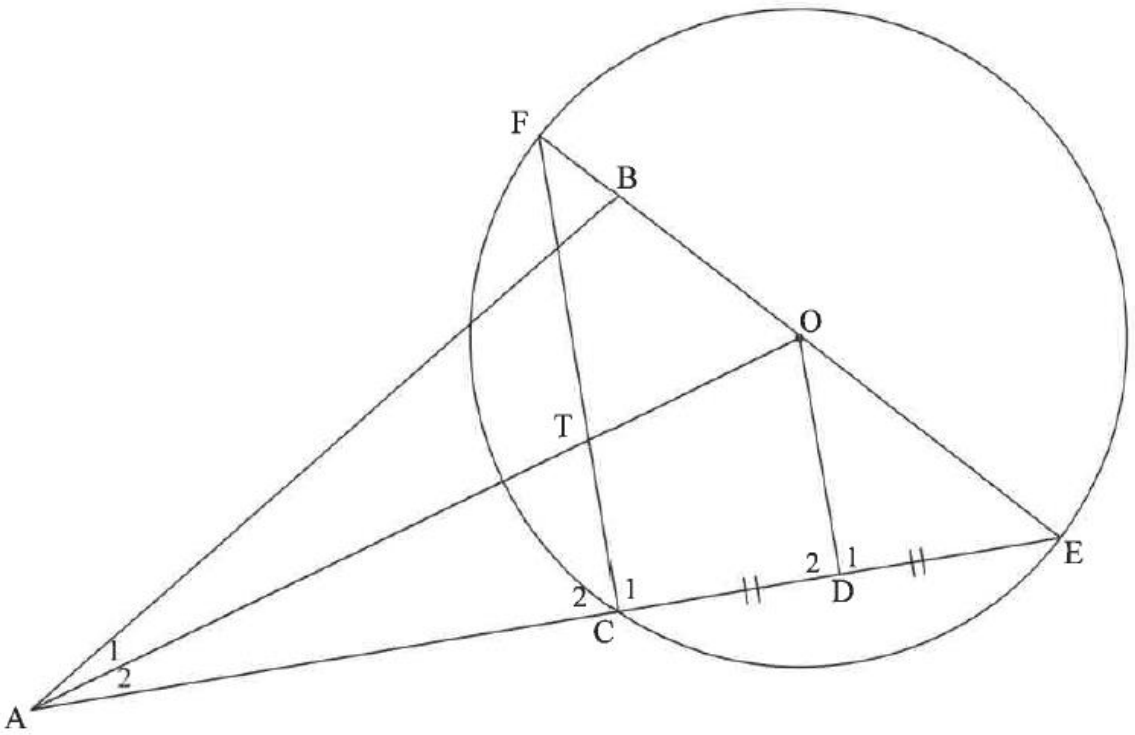
9.2.1 Determine, giving a reason, the value of x. (5)

9.2.2 Determine the ratio: $\frac{\text{area } \Delta RFH}{\text{area } \Delta RST}$. (4)

[14]

QUESTION 10

In the diagram, FBOE is a diameter of a circle with centre O. Chord EC produced meets line BA at A, outside the circle. D is the midpoint of CE. OD and FC are drawn. AFBC is a cyclic quadrilateral.



10.1 Prove, giving reasons, that:

10.1.1 $FC \parallel OD$ (5)

10.1.2 $D\hat{O}E = B\hat{A}E$ (4)

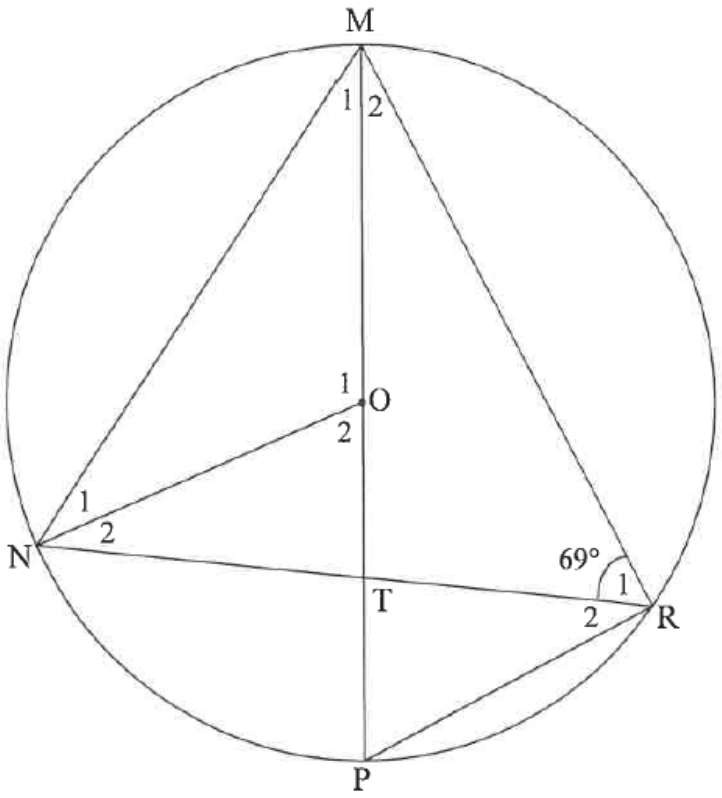
10.1.3 $AB \times OF = AE \times OD$ (7)

10.2 If it is further given that $AT = 3TO$, prove that $5CE^2 = 2BE.FE$ (5)

[21]

QUESTION 8

8.1 In the diagram, MP is a diameter of a circle centered at O. MP cuts the chord NR at T. Radius NO and chords PR, MN and MR are drawn. $\hat{R}_1 = 69°$.



Determine, giving reasons, the size of:

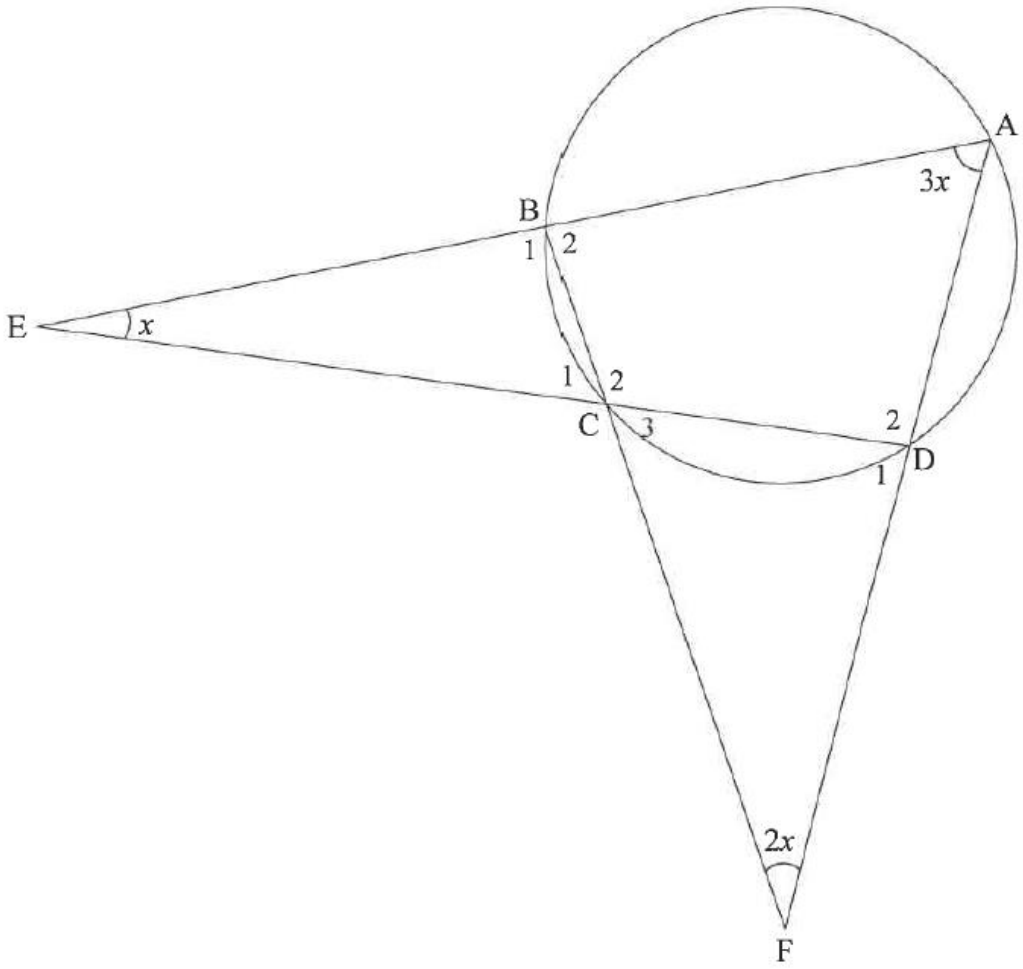
8.1.1 $\hat{R}_2$ (2)

8.1.2 $\hat{O}_1$ (2)

8.1.3 $\hat{M}_1$ (2)

8.1.4 $\hat{M}_2$, if it is further given that NO || PR (4)

8.2 In the diagram below, ABCD is a cyclic quadrilateral. AB and DC are produced to meet at E. AD and BC are produced to meet at F. $A\hat{F}B = 2x$, $D\hat{A}B = 3x$ and $A\hat{E}D = x$.

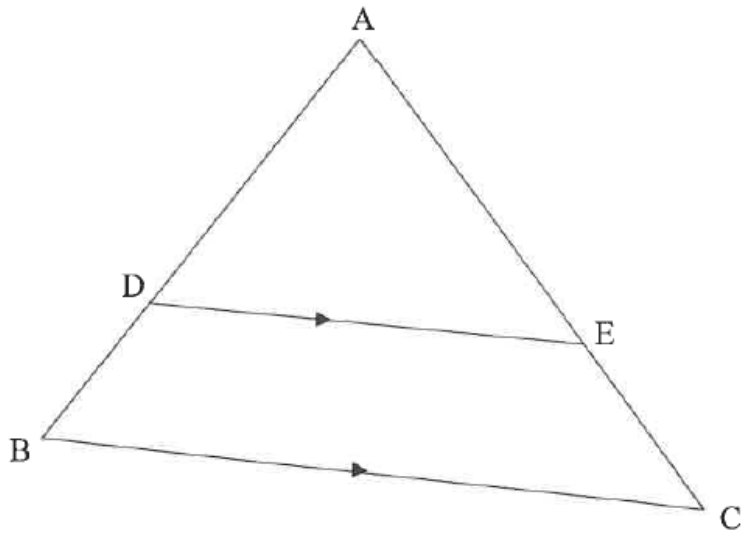


Determine, giving reasons, the value of x. (6)

[16]

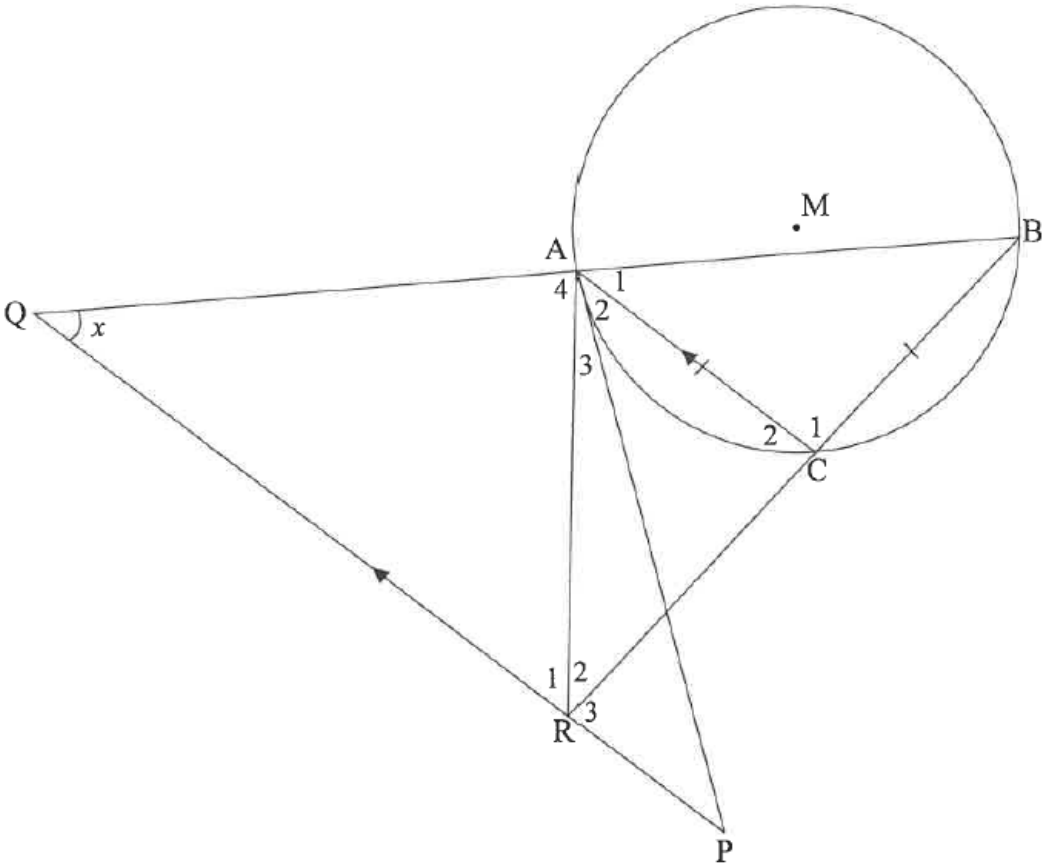
QUESTION 9

9.1 In the diagram, ABC is a triangle. D and E are points on sides AB and AC respectively such that DE || BC .



Use the diagram above to prove the theorem which states that a line drawn parallel to one side of a triangle divides the other two sides proportionally, i.e. prove that $\frac{AD}{DB} = \frac{AE}{EC}$. (6)

9.2 In the diagram, M is the centre of the circle. A, B and C are points on the circle such that AC = BC. PA is a tangent to the circle at A. PQ is drawn parallel to CA to meet BA produced at Q. BC produced meets PQ at R and AR is drawn.
Let $\hat{Q} = x$.



9.2.1 Determine, giving reasons, FOUR other angles EACH equal to x. (6)

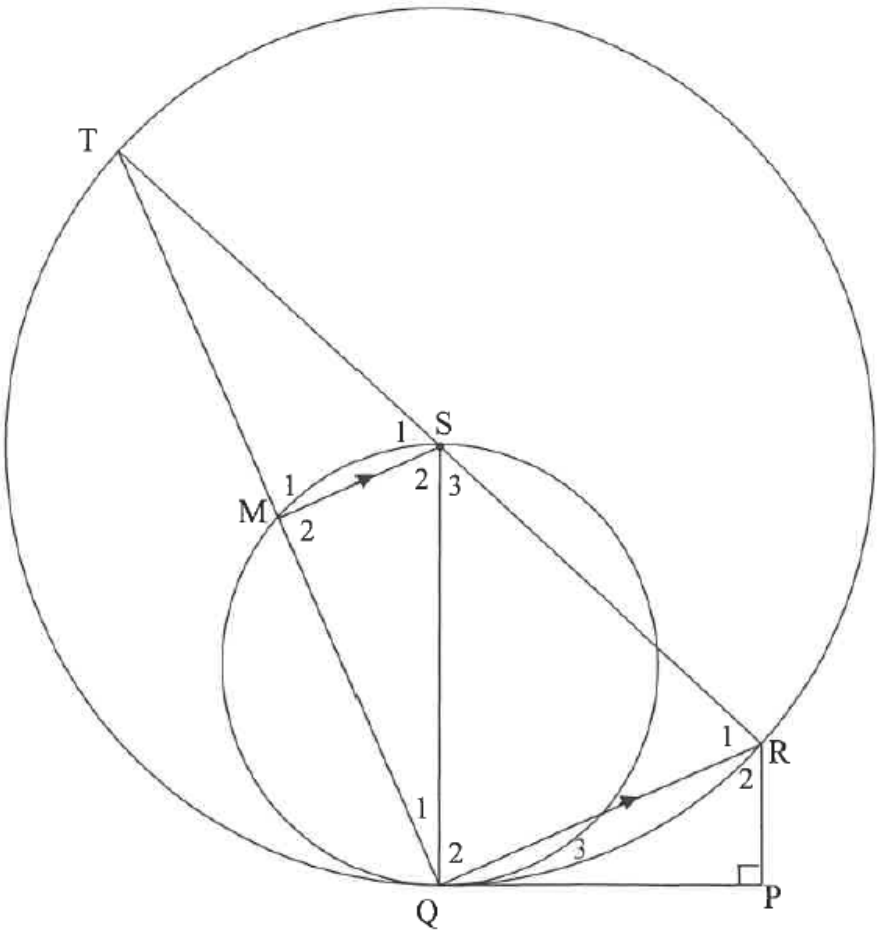
9.2.2 Prove that ABPR is a cyclic quadrilateral. (2)

9.2.3 Prove that $\frac{BA}{BQ} = \frac{BC}{QR}$. (3)

[17]

QUESTION 10

In the diagram, TSR is a diameter of the larger circle having centre S. Chord TQ of the larger circle cuts the smaller circle at M. PQ is a common tangent to the two circles at Q. SQ is drawn.
RP ⊥ PQ and MS || QR.



10.1 Prove, giving reasons that:

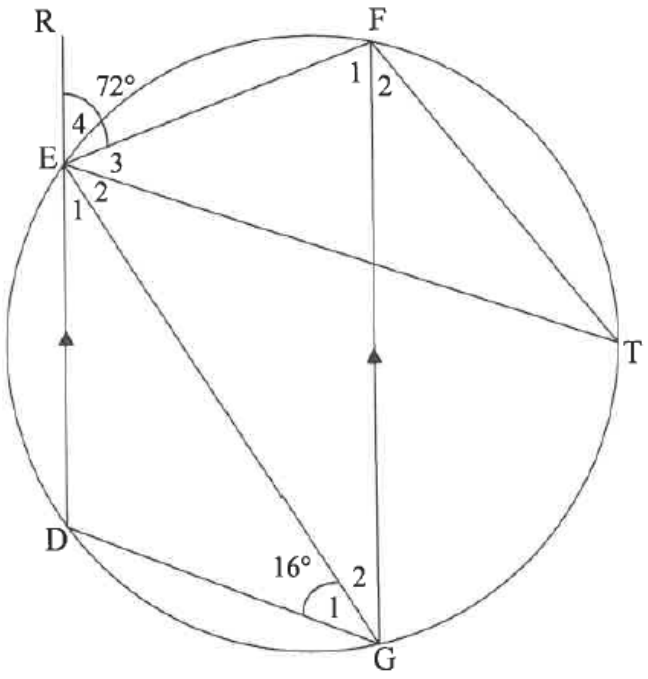
10.1.1 SQ is the diameter of the smaller circle (3)

10.1.2 $RT = \frac{RQ^2}{RP}$ (6)

10.2 If MS = 14 units and $PQ = \sqrt{640}$ units, calculate, giving reasons, the length of the radius of the larger circle. (6)
[15]

QUESTION 9

9.1 In the diagram, DEFG is a cyclic quadrilateral with DE || GF. DE is produced to R. T is another point on the circle. EG, FT and ET are drawn. $\hat{E}_4 = 72°$ and $\hat{G}_1 = 16°$.



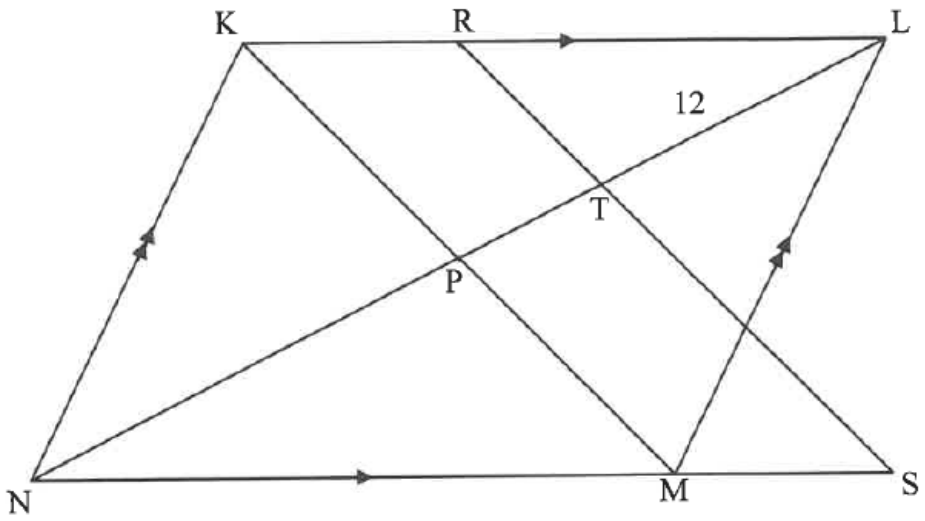
Determine, with reasons, the size of the following angles:

9.1.1 $D\hat{G}F$ (2)

9.1.2 $\hat{T}$ (2)

9.1.3 $G\hat{E}F$ (2)

9.2 In the diagram, the diagonals of parallelogram KLMN intersect at P. NM is produced to S. R is a point on KL and RS cuts PL at T.
NM : MS = 4 : 1, NL = 32 units and TL = 12 units.



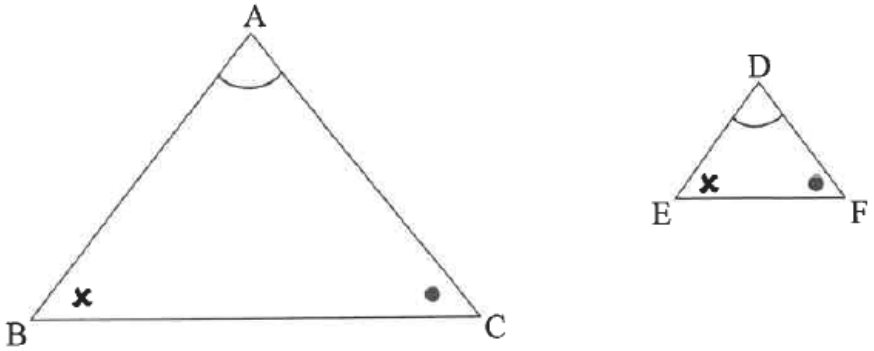
9.2.1 Determine, with reasons, the value of the ratio NP : PT in simplest form. (4)

9.2.2 Prove, with reasons, that KM || RS. (2)

9.2.3 If NM = 21 units, determine, with reasons, the length of RL. (4)
[16]

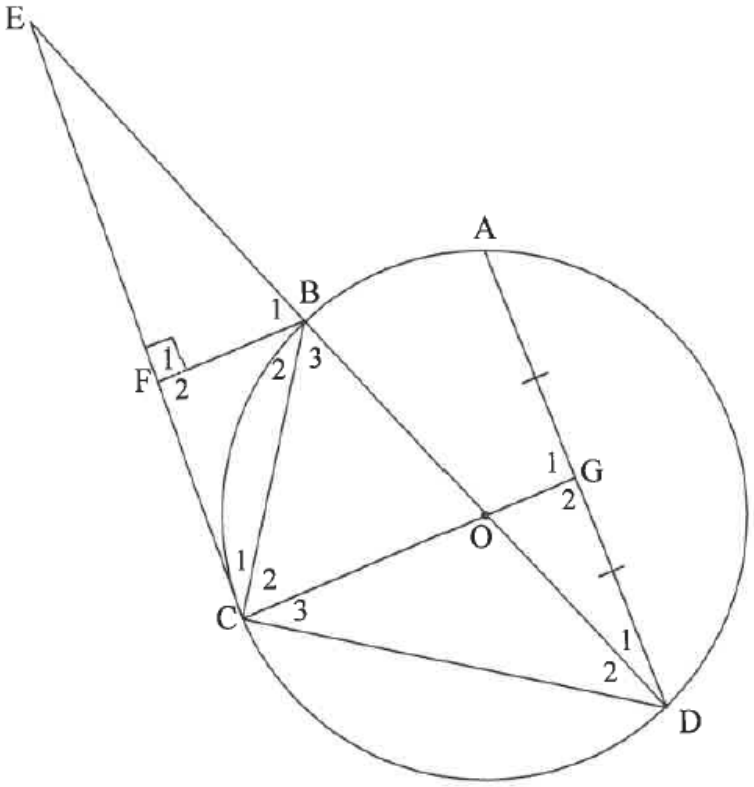
QUESTION 10

10.1 In the diagram, ΔABC and ΔDEF are drawn such that $\hat{A}=\hat{D}$, $\hat{B}=\hat{E}$ and $\hat{C}=\hat{F}$.



Use the diagram in the ANSWER BOOK to prove the theorem which states that if two triangles are equiangular, then the corresponding sides are in proportion, i.e. $\frac{AB}{DE}=\frac{AC}{DF}$. (6)

10.2 In the diagram, O is the centre of a circle passing through A, B, C and D. EC is a tangent to the circle at C. Diameter DB produced meets tangent EC at E. F is a point on EC such that BF $\perp$ EC. Radius CO produced bisects AD at G. BC and CD are drawn.



10.2.1 Prove, with reasons, that:

(a) FB || CG (3)

(b) $\Delta FCB \;|||\; \Delta CDB$ (5)

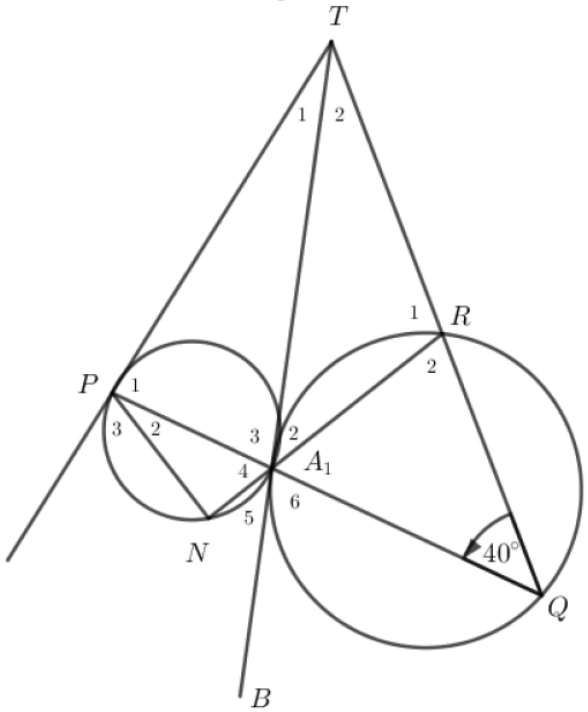
10.2.2 Give a reason why $\hat{G}_1 = 90°$. (1)

10.2.3 Prove, with reasons, that $CD^2 = CG.DB$. (5)

10.2.4 Hence, prove that DB = CG + FB. (5)
[25]

QUESTION 10

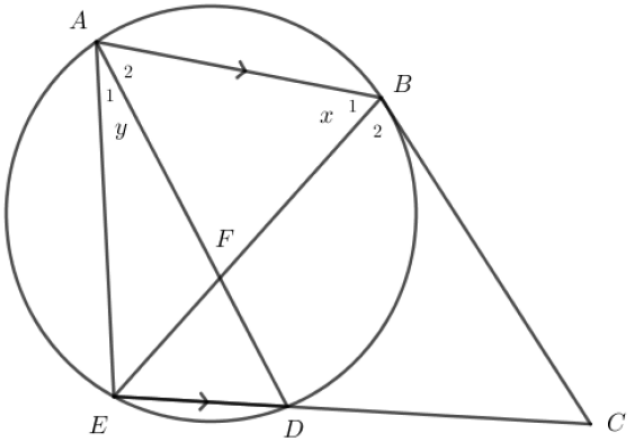
10.1 In the diagram below, two circles have a common tangent TAB. PT is a tangent to the smaller circle. PAQ, QRT and NAR are straight lines. Let $\hat{Q} = 40°$.



10.1.1 Determine, with reasons, TWO other angles each equal to 40°. (4)

10.1.2 If $\hat{P}_1 = \hat{A}_4$ prove that PTRN is a parallelogram. (5)

10.2 Tangent BC touches circle ABDE at B. Chords AD and BE intersect at F. Chord ED is produced to C with AB || EC. It is further given that $\hat{B}_1 = x$ and $\hat{A}_1 = y$.



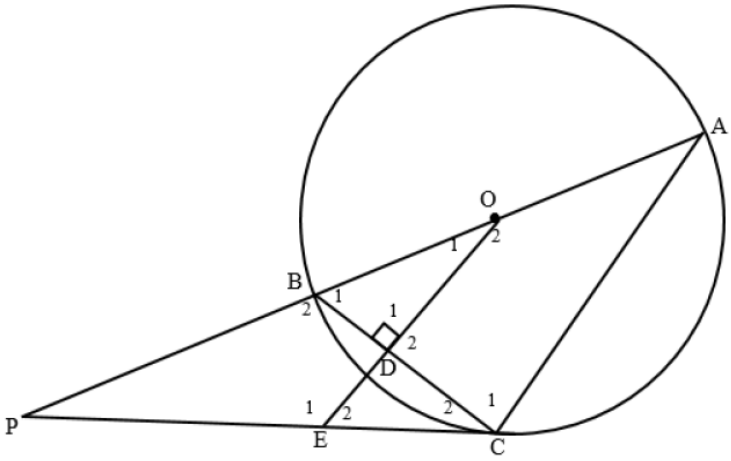
10.2.1 Determine the size of $\hat{C}$ in terms of x and y. (6)

10.2.2 State whether BCDF is a cyclic quadrilateral or not, giving reasons for your answer. (2)

[17]

QUESTION 11

In the diagram, AB is a diameter of circle, centre O. AB is produced to P.
PC is a tangent to the circle at C. OE intersects BC at D such that OE ⊥ BC.



11.1 Prove, with reasons, that EO ∥ CA. (4)

11.2 If $\hat{C}_2 = x$, name with reasons TWO other angles each equal to x. (3)

11.3 Determine the size of $\hat{P}$ in terms of x. (3)

[10]

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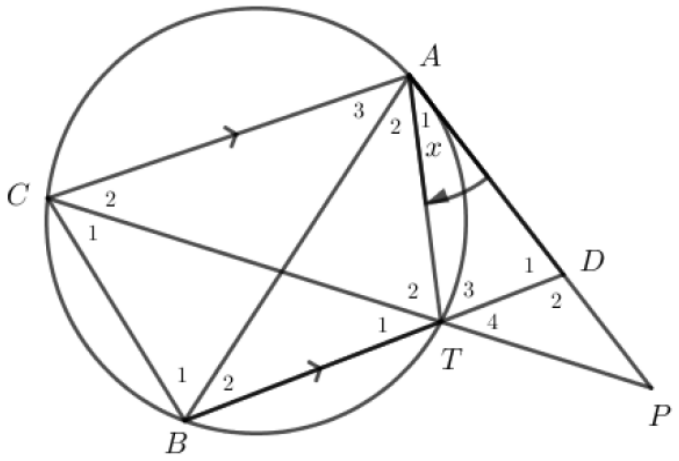
QUESTION 12

In the diagram below DA is a tangent to the circle ACBT at A. CT and AD are produced to meet at P. BT is produced to cut PA at D. AC, CB, AB and AT are drawn. AC is parallel to BD. Let $\hat{A}_1 = x$.



12.1 Find the size of T_1 in terms of x. (3)

12.2 Hence, prove that PT is a tangent to the circle ADT at T. (3)

12.3 Prove that $\Delta APT \,|||\, \Delta TPD$ (3)

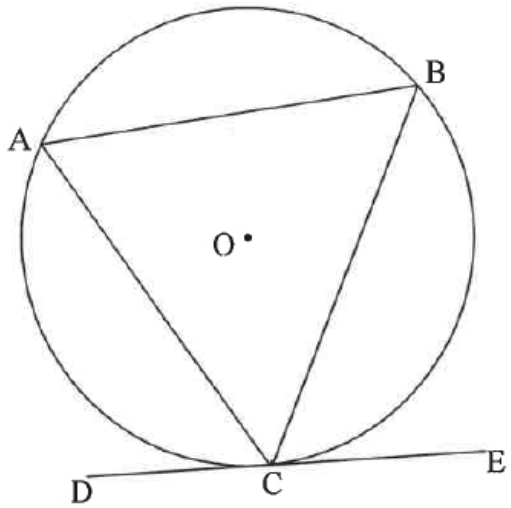
12.4 Find the length of PT if AP = 9 units and PD = 4 units. (3)

[12]

DBE/2024

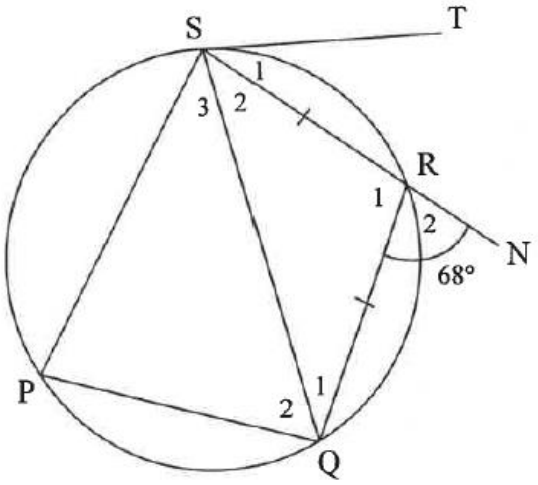
QUESTION 8

8.1 In the diagram, chords AB, BC and AC are drawn in the circle with centre O. DCE is a tangent to the circle at C.



Prove the theorem which states that the angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment, i.e. $B\hat{C}E = \hat{A}$. (5)

8.2 In the diagram, PQRS is a cyclic quadrilateral with RQ = RS. ST is a tangent to the circle at S. SR is produced to N. $\hat{R}_2 = 68°$.



Determine, with reasons, the size of:

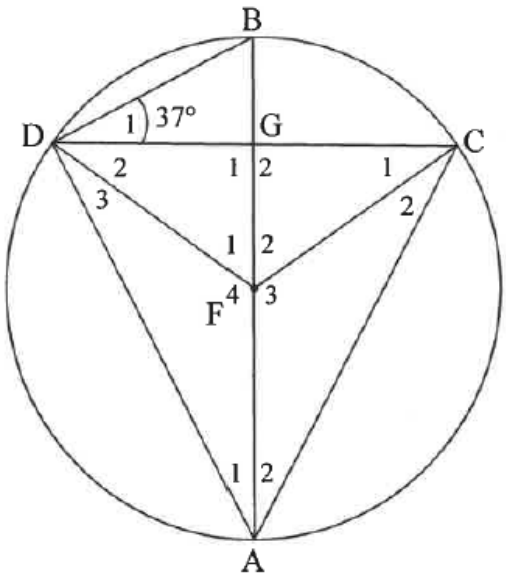
8.2.1 $\hat{P}$ (2)

8.2.2 $\hat{Q}_1$ (2)

8.2.3 $\hat{S}_1$ (2)
[11]

QUESTION 9

In the diagram, AB is a diameter of the circle, with centre F. AB and CD intersect at G. FD and FC are drawn. BA bisects $C\hat{A}D$ and $\hat{D}_1 = 37°$.



9.1 Determine, giving reasons, any three other angles equal to $\hat{D}_1$. (4)

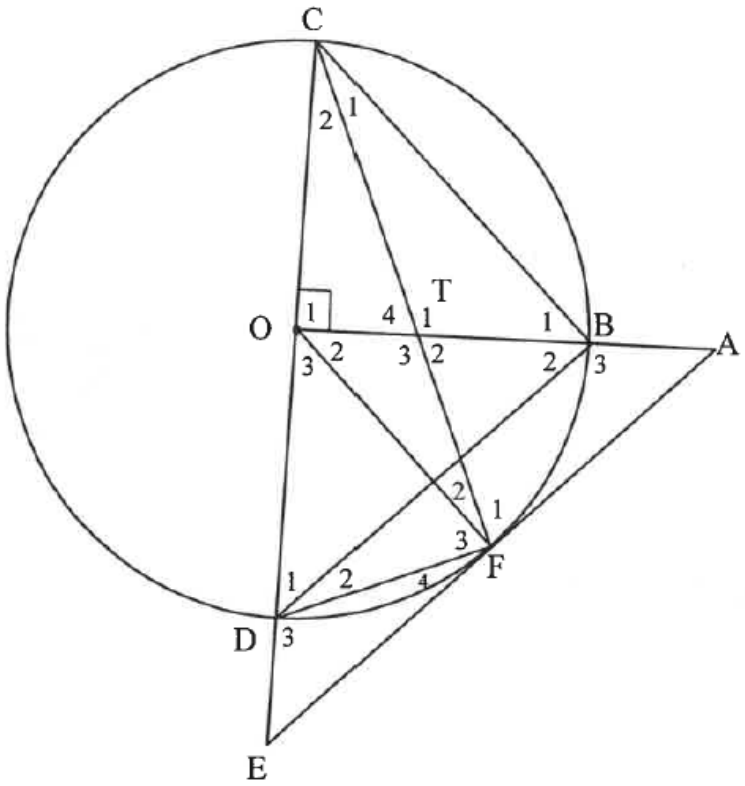
9.2 Show that DG = GC. (4)

9.3 If it is further given that the radius of the circle is 20 units, calculate the length of BG. (4)

[12]

QUESTION 10

In the diagram, COD is the diameter of the circle with centre O. EA is a tangent to the circle at F. AO ⊥ CE. Diameter COD produced intersects the tangent to the circle at E. OB produced intersects the tangent to the circle at A. CF intersects OB in T. CB, BD, OF and FD are drawn.



Prove, with reasons, that:

10.1 TODF is a cyclic quadrilateral (4)

10.2 $\hat{D}_3 = \hat{T}_1$ (3)

10.3 $\Delta TFO \;|||\; \Delta DFE$ (5)

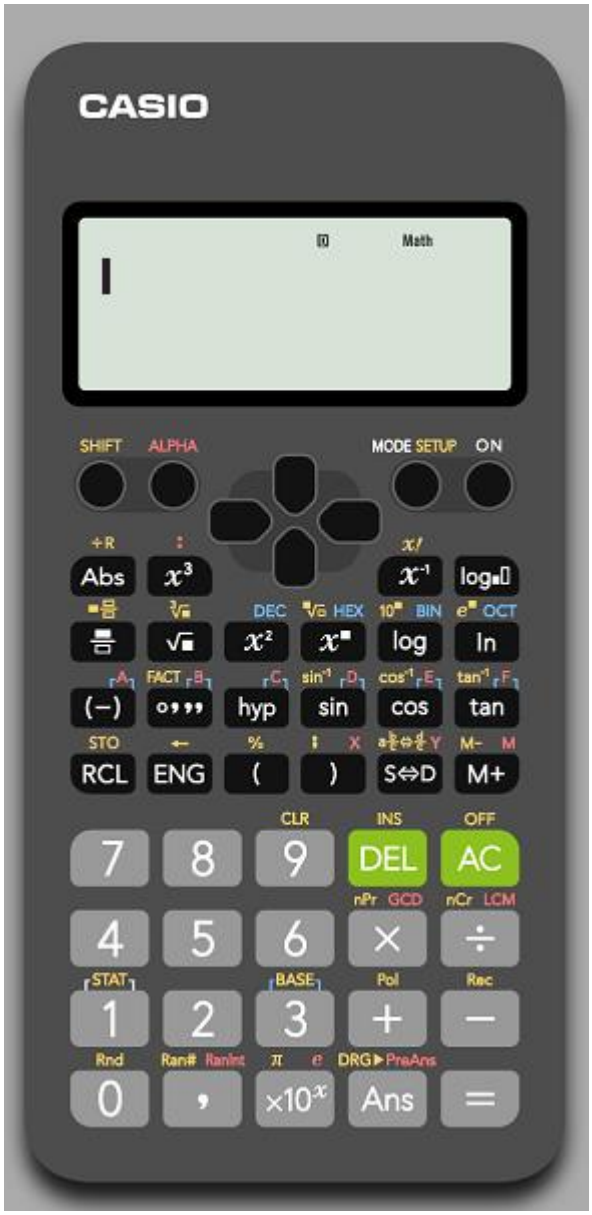
10.4 If $\hat{B}_2 = \hat{E}$, prove that DB || EA. (2)

10.5 Prove that $DO = \dfrac{TO.FE}{AB}$ (5)

[19]

END

FX82ZA+ II General Calculator Workbook
By Lauren Izaaks



MODE

```
1:COMP    2:STAT
3:TABLE   4:BASE-N
5:RATIO
```

SHIFT

Selects second options

ALPHA

Selects variables or third options

Important Tips and Tricks 😊

Resetting/ Clearing the Calculator:

SHIFT 9

```
Clear?
1:Setup  2:Memory
3:All
```

3

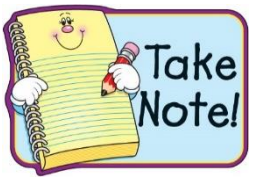
```
Reset All?

[=]    :Yes
[AC]   :Cancel
```

=

```
    Reset All

 Press [AC] key
```



There is a difference between OFF AC and ON

Let's look at some of the basics:

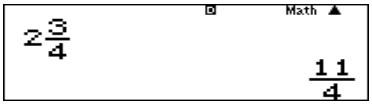
Common Fractions:

Simplify the following:

$\frac{10}{25}$ = ----- Key Sequence: [▤] [1] [0] [▼] [2] [5] [=]

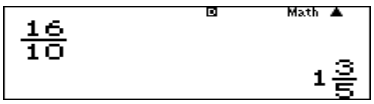
Mixed Fractions:

$2\frac{3}{4}$ = ----- Key Sequence: [SHIFT] [▤] [2] [▶] [3] [▼] [4] [=]



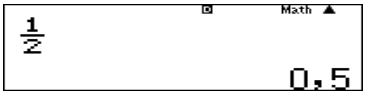
Improper Fractions:

$\frac{16}{10}$ = ----- Key Sequence: [▤] [1] [6] [▼] [1] [0] [=] [SHIFT] [S⇔D]



Decimals:

$\frac{1}{2}$ = ----- Key Sequence: [▤] [1] [▼] [2] [=] [S⇔D]



Let's put this together:

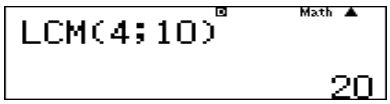
Mixed Fraction	Improper Fraction	Decimal
$3\frac{2}{3}$		
	$\frac{5}{18}$	
		0,6

Greatest Common Divisor (GCD) & Lowest Common Multiple (LCM):

Greatest Common Divisor (GCD)= Highest Common Factor (HCF)

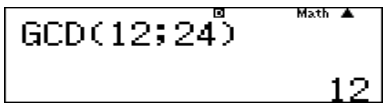
Find the LCM of 4 and 10 _____

Key Sequence: [ALPHA] [÷] [4] [SHIFT] [)] [1] [0] [)] [=]



Find the GCD of 12 and 24 _____

Key Sequence: [ALPHA] [×] [1] [2] [SHIFT] [)] [2] [4] [)] [=]

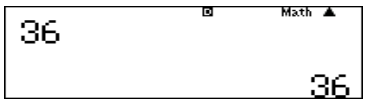


You are only able to find the GCD and LCM of 2 Numbers!

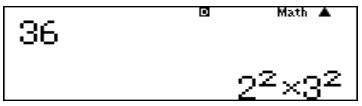
Prime Factors:

Find the Prime Factors of 36 _____

Step 1: Key Sequence [3] [6] [=]



Step 2: Key Sequence [SHIFT] [°''']



Now Try This:

Find the Prime Factors of 101 _____

Why does this happen?? _____

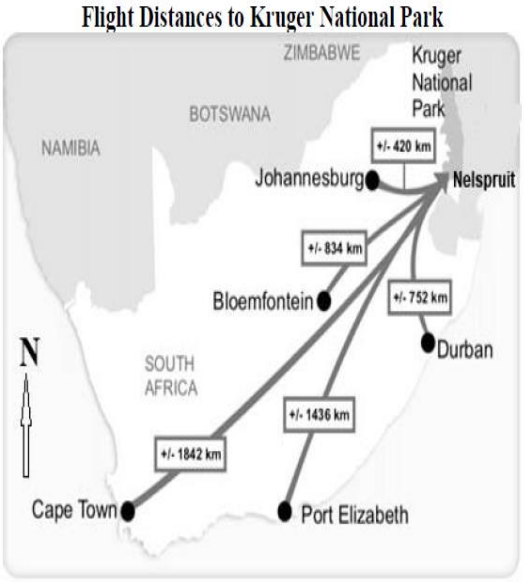
_____

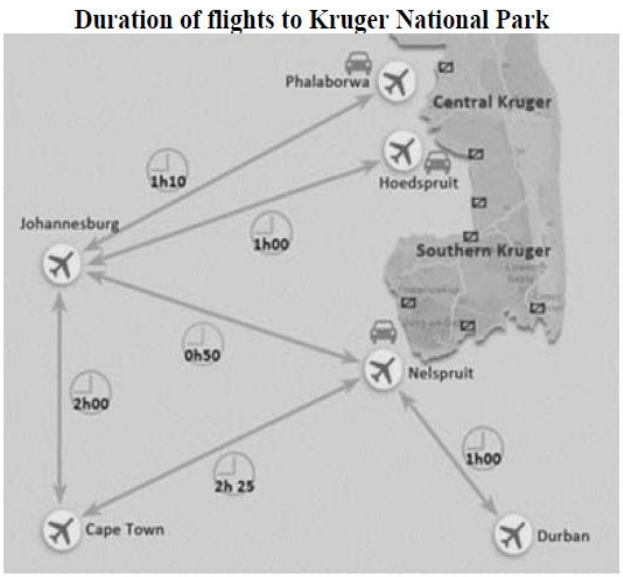
Time Calculations:

Rule 1: Always work in Hours, Minutes and Seconds!

Rule 2: Always remember to push the Time Button after every Hour, Minute and Second!

Look at the pictures below and answer the questions that follow





Adding/ Subtracting Times:

A businessman needs to be at the Kruger Park for a meeting at 13:30. He is first meeting his business partner for two hours at OR Tambo International Airport (Johannesburg).

Will the businessman be on time for his meeting if his flight is at 08:20 from Cape Town and then again at 12:55 from OR Tambo? (Show all calculations).

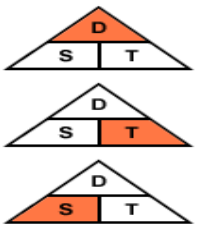
Calculation:	Key Sequence:
Cape Town to OR Tambo 08:20 + 2 Hours =_____	0 8 °‘‘‘ 2 0 °‘‘‘ 0 0 °‘‘‘ + 0 2 °‘‘‘ 0 0 °‘‘‘ 0 0 °‘‘‘ =
2 Hour Meeting at OR Tambo 10:20 + 2 Hours = _____	1 0 °‘‘‘ 2 0 °‘‘‘ 0 0 °‘‘‘ + 0 2 °‘‘‘ 0 0 °‘‘‘ 0 0 °‘‘‘ =
OR Tambo to Nelspruit 12:55 + 50 Minutes = _____ Therefore _____	1 2 °‘‘‘ 5 5 °‘‘‘ 0 0 °‘‘‘ + 0 0 °‘‘‘ 5 0 °‘‘‘ 0 0 °‘‘‘ =

Rate:

Determine the speed that the aeroplane is travelling at from Cape Town to Nelspruit. Give your final answer to the nearest kilometer per hour. (Use the pictures provided above)

You may use the following formula:

Distance= Speed x Time



Distance = Speed x Time

Time= $\frac{\text{Distance}}{\text{Speed}}$

Speed= $\frac{\text{Distance}}{\text{Time}}$

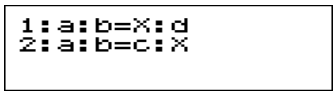
Calculation:	Key Sequence:
Distance = Speed x Time	
1 842km = Speed x 2 Hrs 25 mins	
Convert 2 Hours 25 Minutes into Hours (Decimal)	0 2 °‘‘‘ 2 5 °‘‘‘ 0 0 °‘‘‘ = S⇔D S⇔D 02°25°00° Math 2,416666667
$S = \frac{1842}{2,416666667}$ Therefore 762km.	▣ 1 8 4 2 ▼ 2 , 4 1 6 6 6 6 6 6 7 = 1842 / 2,416666667 Math 762,2068964

Ratio:

What can I use Ratio Mode for?	
Calculating equivalent fractions	Work out the cost of different amounts
Write ratios in the form n:1	Find lengths using the sine rule
Finding the lengths of similar triangles	Working out class marks

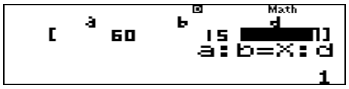
Eg 1: I have 60 Red Balls and 15 Blue Balls. Write this in the simplest ratio x:1.

Step 1: Go into Ratio Mode MODE 5

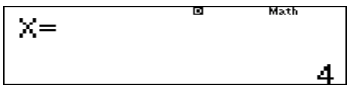


Step 2: Select Option 1

Step 3: Enter in the values given. 6 0 = 1 5 = 1 =



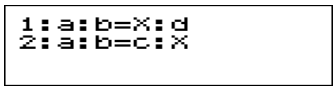
Step 4: Remember to check the values are correct then Press =



Therefore 60:15= 4:1

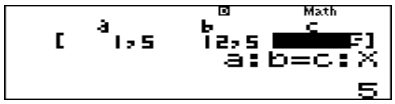
Eg 2: If 1,5kg of potatoes cost me R12.50. How much will 5kg cost?

Step 1: Go into Ratio Mode MODE 5

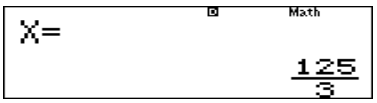


Step 2: Choose Option 2

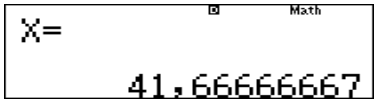
Step 3: Enter in the values 1 , 5 = 1 2 , 5 0 = 5 =



Step 4: Remember to check the values are correct then Press =



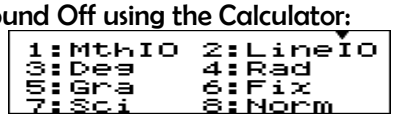
Step 5: Press S⇔D



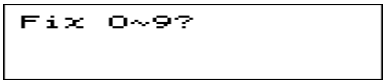
This is Financial Maths and therefore final answers need to be correct to 2 Decimal Places.

Let's see how we can Round Off using the Calculator:

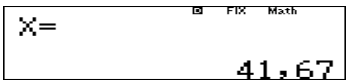
Step 1: SHIFT MODE



Step 2: Select 6 : Fix



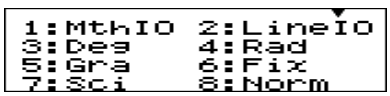
Step 3: Choose how many decimal places your answer is rounded off to. Press 2 and S⇔D



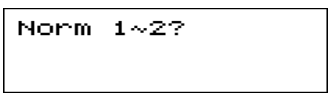
Note the word Fix on the screen of the calculator. This indicates that all answers are rounded to 2 decimal places from now on.

NB: Go back to Normal when finished rounding off!!

SHIFT MODE



8



Option 1- Physics (Scientific Notation) and Option2- Maths (Decimals) Choose 2

Tables Mode:

Completing a Table:

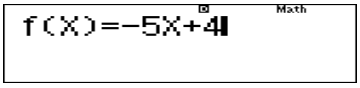
Find the ordered pairs of the given equation by completing the table and then draw the graph.

F(x)= -5x + 4

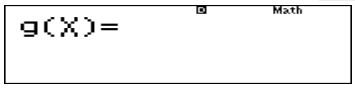
x	F(x)	(x ; y)
-1		
0		
1		
2		

Step 1: Go into Tables Mode [MODE] [3]

Step 2: Enter in the function given. [−] [5] [ALPHA] [)] [+] [4]



Step 3: Press [=]



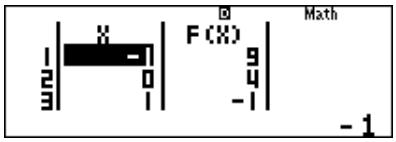
What is a g(x)= _____

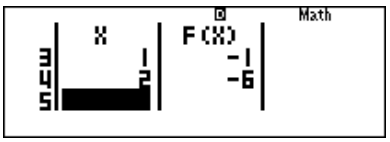
Step 4: Press [=]

Step 5: Refer to the Table provided. Start at [−] [1] Press [=]

Step 6: Refer to the Table provided. End at [2] Press [=]

Step 7: Refer to the Table provided. Step in [1] Press [=]





We now have the F(x) and therefore can complete the Table.

Notes:

Where do I Start?

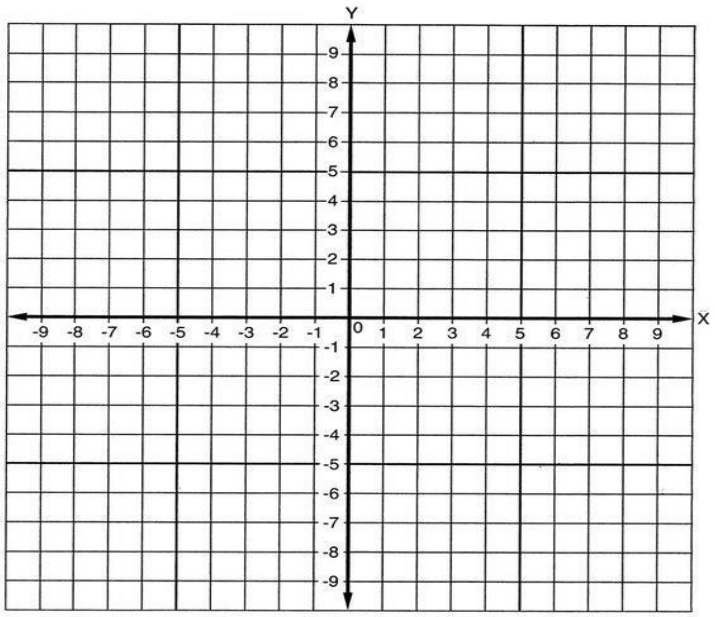
Start at the Lowest X-Value.

Where do I End?

End at the Highest X-Value.

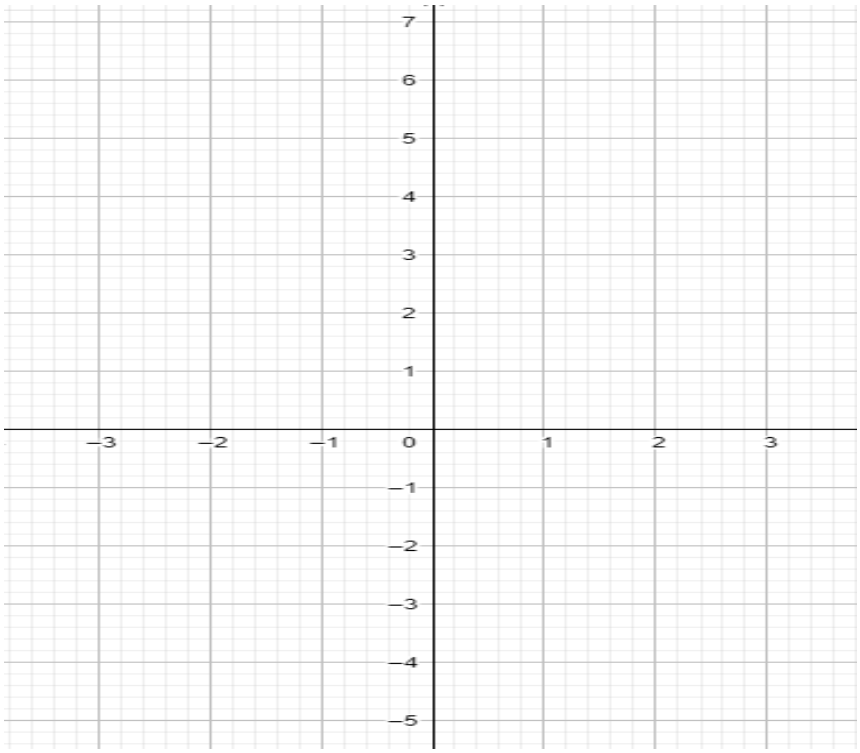
What is a Step?

Step= Intervals (How much do I increase up by every time I take a step)



Drawing 2 Graphs on the same set of axes:

Draw the graphs of $F(x)= x^2 - 2$ and $G(x)= 2x + 1$ on the same set of axes.



Follow the same steps as above.

Step 1: Go into Tables Mode [MODE] [3]

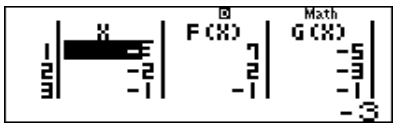
Step 2: Enter in the f(x) function [ALPHA] [)] [x^2] [−] [2] Press [=]

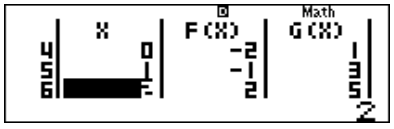
Step 3: Enter in the g(x) function [2] [ALPHA] [)] [+] [1] Press [=]

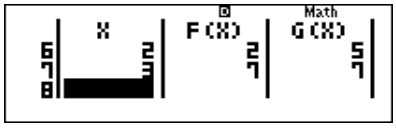
Step 4: Select the Start Point. _____ Press [=]

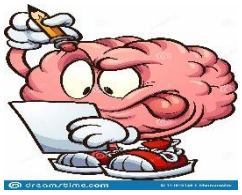
Step 5: Select the End Point. _____ Press [=]

Step 6: Choose the Step. _____ Press [=]









What can I find from this information??

y- intercepts f(x): (0 ; -2), g(x): (0 ; 1)

Points of intersection f(x) = g(x): (-1 ; -1), (3 ; 7)

Turning Point f(x): (0 ; -2)

Solving Quadratic Equations and Factorising:

Solve the following equation $x^2 + 2x - 3 = 0$

Step 1: Go into Tables Mode [MODE] [3]

Step 2: Enter in the Equation (make sure everything is on one side and equal to Zero)

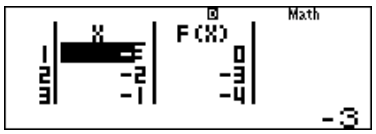
[ALPHA] [)] [x^2] [+] [2] [ALPHA] [)] [−] [3] Press [=]

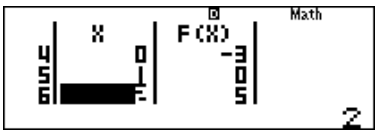
Step 3: There is no second equation to solve therefore Press [=]

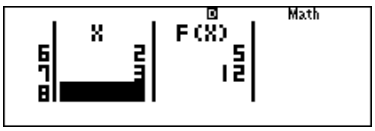
Step 4: The Start will be the Negative value of the Constant [−] [3] Press [=]

Step 5: The End will be the Positive value of the Constant [3] [=] Press [=]

Step 6: Always step in 1's, if the answer is fractions/ decimals learners must use the Quadratic Formula to show working out. [1] Press [=]







Look for where Y equals Zero. (-3 ; 0) and (1 ; 0).

Therefore x= -3 or x= 1.

If asked to factorise, remember to put this into brackets and change the signs $(x + 3)(x - 1)$.

Another Example:

Solve the following equation $x^2 + 3x + 2 = 0$

Step 1: Go into Tables Mode [MODE] [3]

Step 2: Enter the Equation [ALPHA] [)] [x^2] [+] [3] [ALPHA] [)] [+] [2] Press [=]

Step 3: There is no second Equation so Press [=]

Now complete it on your own...

Step 4: Start _____

Step 5: End _____

Step 6: Step _____

Therefore x= _____ or x= _____.

See, it's not so difficult!

Statistics:

Single Variable Data Handling:

The following list of data shows the number of runs a cricket player has made over the past 10 matches played: 56, 39, 40, 35, 56, 67, 37, 45, 36, 60.

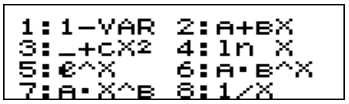
Answer the questions that follow:

a) How many runs did he make in total across all 10 matches played?
b) What is his average number of runs scored in a match?
c) What is the highest number of runs he scored?

Focus will be on:

1) Data Handling
2) Linear Regression

Step 1: Go into Stats Mode [MODE] [2]



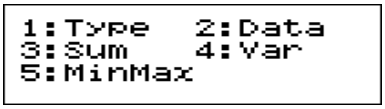
Step 2: Select Option [1] (Single Variable Data Handling)

Step 3: Enter in the data given (Order does not matter)

[5] [6] [=] [3] [9] [=] [4] [0] [=] [3] [5] [=] [5] [6] [=] [6] [7] [=]
[3] [7] [=] [4] [5] [=] [3] [6] [=] [6] [0] [=]

Always check the data before continuing!

Step 4: [AC] [SHIFT] [1]



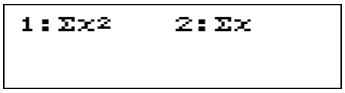
Stats Menu Explained:

Key	Menu Item	Explanation
1. Type	Stats Menu	Changes stats type
2. Data		Displays the data that you input
3. Sum	1. Σx^2 2. Σx	1. Sum of the squares 2. Sum/ Total of data
4. Var	1. n 2. $\bar{x}$ 3. δx 4. $s x$	Number of samples Mean (Average) Population standard deviation Sample standard deviation
5. MinMax	1. Min 2. Max	1. Indicates the minimum value 2. Indicates the maximum value

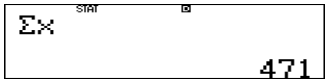
Answer the Questions:

How many runs did he make in total?

Step 1: Go into [3]: Sum



Step 2: Select [2] then Press [=]

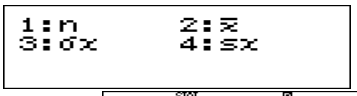


Total Runs= 471.

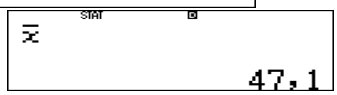
What is his average number of runs?

Step 1: Go back to the Stats Menu [AC] [SHIFT] [1]

Step 2: Select [4]: Var



Step 3: Select [2] and Press [=]

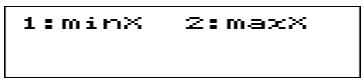


Mean/ Average= 47,1

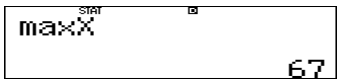
What is his highest score?

Step 1: Go back into the Stats Menu [AC] [SHIFT] [1]

Step 2: Select [5]: MinMax



Step 3: Select [2] and Press [=]



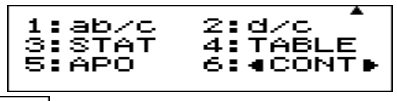
Highest/ Maximum Runs= 67.

Grouped Data:

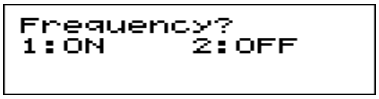
30 People were asked what percentage of their income they spent on eating out every month. The following table shows their responses:

Percentage	Frequency
10 ≤ x < 20	7
20 ≤ x < 30	8
30 ≤ x < 40	6
40 ≤ x < 50	5
50 ≤ x < 60	4

Step 1: Switch a Frequency Table on [SHIFT] [MODE] [▼]



Choose [3]: Stat



Choose [1]: On

Remember to Switch the Frequency Table off when you are not using it!

Step 2: Calculate the midpoints of the grouped data and enter this, followed by the Frequency.

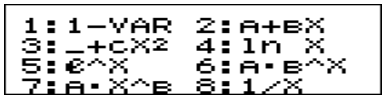
[1] [5] [=] [2] [5] [=] [3] [5] [=] [4] [5] [=] [5] [5] [=] [▶] [▼] [7] [=] [8] [=] [6] [=] [5] [=] [4] [=]

Linear Regression:

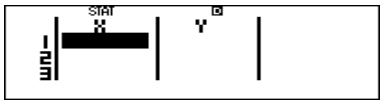
A sales manager collected the following information on the number of sales calls made and the number of copiers sold by 5 of his representatives.

Sales Representative	No. of Calls Made	No. of Copiers Sold
John Smith	20	30
Jackie Burns	40	60
Theresa Lee	20	40
David Jones	30	60
James Du Preez	10	30

Step 1: Go into Stats Mode [MODE] [2]



Step 2: Select Option [2]

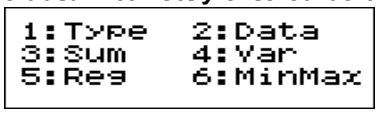


Step 3: Enter the data (Enter the X- Values first, then go back to the top and enter the Y- Values)

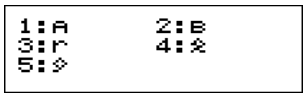
[2] [0] [=] [4] [0] [=] [2] [0] [=] [3] [0] [=] [1] [0] [=] [▼] [▶]
[3] [0] [=] [6] [0] [=] [4] [0] [=] [6] [0] [=] [3] [0] [=]

Always remember to check the data is correctly entered before continuing.

Step 4: Press [AC] [SHIFT] [1]



Select [5]: Reg



Regression Menu Explained:

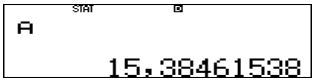
Menu Item	Explanation
1. A	Regression co- efficient of A
2. B	Regression co- efficient of B
3. r	Correlation co- efficient
4. $\hat{x}$	Estimated value of x
5. $\hat{Y}$	Estimated value of y

Answer the questions that follow:

1) Calculate the Line of Best Fit

Y= A + Bx

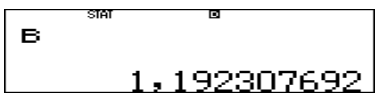
[1] [=]



A= 15,38

Now find B

[AC] [SHIFT] [1] [5] [2] [=]

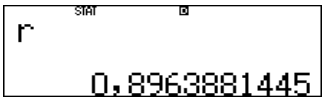


B= 1,19

Therefore y= 15,38 + 1,19x

2a) Calculate the Correlation Co- efficient

AC SHIFT 1 5 Choose Option 3 :Reg and Press =



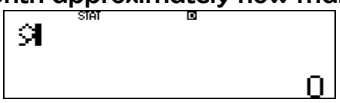
r = 0,90

2b) Describe the Correlation

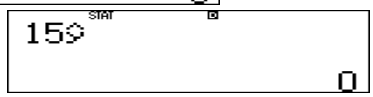
A strong positive correlation

3) If a sales person makes 15 calls in one month approximately how many copiers will they sell?

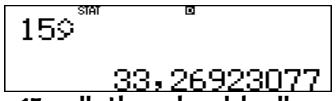
AC SHIFT 1 5 Choose Option 5 : $\hat{Y}$



Move the curser to the front ◀ and enter 1 5



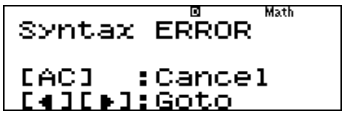
Press =



If they make 15 calls they should sell approximately 33 copiers.

A few extra notes:

Syntax Error



What does this mean? How do I clear it?
You have entered a sign or symbol incorrectly or left a sign or symbol out.
Do not Press AC rather use your arrows and Go To the mistake 😊

Math Error

What does this mean?
Your answer is Undefined

For more calculator resources visit our Website:
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